29/8/'16 ELL 700 Linear System Theory

MINOR TEST I   Marks: 20, Time: 1 hr

Q1:- (a) Consider a linear system with input u & output y. Three experiments are performed on the system using inputs $u_1(t)$, $u_2(t)$ & $u_3(t)$ for $t \geq 0$. In each case, the initial state $X(0)$ at time $t=0$ is the same. The corresponding outputs are denoted by $y_1(t)$, $y_2(t)$ & $y_3(t)$. WHICH of following statements are correct if $X(0) \neq 0$

S1: If $u_3 = u_1 + u_2$ then $y_3 = y_1 + y_2$

S2: If $u_3 = 0.5(u_1 + u_2)$ then $y_3 = 0.5(y_1 + y_2)$

S3: If $u_3 = u_1 - u_2$ Then $y_3 = y_1 - y_2$

WHICH statements are correct if $X(0) = 0$. Give reasons.

2. 1(b) Consider eq^n $X(n) = A^n X(0) + A^{n-1} b u(0) + A^{n-2} b u(1) + \cdots + A b u(n-2) + b u(n-1)$.

where A is $n \times n$ and b is $n \times 1$. Show Under what conditions on A and b will there exist $u(0), u(1), \cdots, u(n-1)$ to meet the eq^n for any $X(n)$ and $X(0)$

2. 1(c) Show whether following statement is TRUE or NOT

det(A) = Product of all eigenvalues of A   (5)

Q2:- (a) Matrix A $n \times n$ has distinct eigenvalues. Let q_i be right eigenvector of A associated with λ_i, $i = 1, 2, \cdots n$. Define $Q = [q_1, q_2, \cdots, q_n]$. Also define $P = Q^{-1} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}$ where p_i is i^{th} row of P.

Show whether p_i is a left eigenvector of A associated with λ_i or not.

Q.2 (b):- It is known that

$$e^{At} = \begin{bmatrix} 2e^{t} - e^{2t} & 2te^{t} & 2e^{t} - 2e^{2t} \\ 0 & e^{t} & 0 \\ e^{2t} - e^{t} & -te^{t} & 2e^{2t} - e^{t} \end{bmatrix}$$

(6)

Find A.

Q.3:- Consider $\dot{x} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda^* \end{bmatrix} x + \begin{bmatrix} b_1 \\ b_1^* \end{bmatrix} u$, $y = [c_1 \; c_1^*] x$

where ~~overbar~~ * denotes complex conjugate

SHOW whether or not this eq^n can be transformed into $\dot{\bar{x}} = \bar{A}\bar{x} + \bar{b}u$, $y = \bar{c}\bar{x}$

with $\bar{A} = \begin{bmatrix} 0 & 1 \\ -\lambda\lambda^* & \lambda+\lambda^* \end{bmatrix}$, $\bar{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\bar{c} = [-2\,Re(\lambda^* b_1 c_1) \; ; \; 2\,Re(b_1 c_1)]$

by using transformation $x = Q\bar{x}$

with $Q = \begin{bmatrix} -\lambda^* b_1 & b_1 \\ -\lambda b_1^* & b_1^* \end{bmatrix}$ (5)

~~Q.4:- Let $\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x$, $x(t=0) = x(0)$~~

~~Find the solution $x(t)$~~

Q.4:- Find the unit step response of

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$ (4)

$$y = [2 \;\; 3] x$$

Note:- Show all steps in your answers.