

ELL 701 - Mathematical Methods in Control

Major: Nov 2016

Instructor: M. Nabi

1. For a matrix A , it is seen that

$$A = U \begin{bmatrix} 7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} V^T$$

where $U = [u_1 \ u_2 \ u_3]$, $V = [v_1 \ v_2 \ v_3]$ and $V^T V = U^T U = I$
Then -

- What is the condition number of A ?
- The distance to the nearest rank-2 matrix is
- Express A as a sum of rank-1 matrices, using only quantities defined in the question.
- For a vector x having 2-norm equal to 5, can the 2-norm of the vector Ax be 50 ?
Why ?
 $1+1+2+2 = 6$

2. On what conditions on $[b_1, b_2, b_3]^T$ is the system solvable? Describe the situation pictorially.

$$\begin{bmatrix} -1 & 2 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$2+2=4$

- Corresponding to a quadratic function $f(x) = 10x_1^2 + 3x_2^2 + 4x_3^2 + 4x_2x_3 + 2x_3x_1$, find the matrix A such that $f(x)$ is a quadratic form corresponding to A .
 - Find if A is positive definite.
 - If the Rayleigh quotient $R(x)$, connected with the matrix A is maximum/minimum at the point x^* , then what is this point x^* , and what is the maximum/minimum value? Say, with the help of proof.

$2+4+4 = 10$

P.T.O.

4. Let $[X_1 X_2]^T$ be a Gaussian random vector with mean $[0 \ 1]^T$ and covariance matrix,

$$C_{XX} = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$$

(a) Write the joint PDF $f_X(x)$. You can give in the vector-matrix form, but explaining the vectors and matrices and giving their values in this case.

(b) If $Y = Ax + B$ with

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}; B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

find C_{YY} .

$$2+3=5$$

5. A random process is such that, $E[X(t)] = 2$ always and $R_{XX}(\tau)$ is equal to $\frac{3}{1+\tau^2}$.

(a) Is the process WSS?

(b) What is $E[X^2(t)]$?

(c) What is $C_{xx}(\tau)$?

$$1+2+2=5$$