ELL 701 - Mathematical Methods in [illegible]

Aug 2016

Name: [illegible]

1. Compute $\|x\|_1$, $\|x\|_2$ and $\|x\|_\infty$ for vector $x = (-4, 2, -3, \ldots)$

2. Verify if the class of all (2×2) matrices having the $(1,1)$ element [illegible] the $(2,2)$ element, form a vector space. If yes, give a basis and the dimension of the space.

3. A linear transformation is represented with respect to the standard basis by a matrix

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

Now the basis is shifted to the vectors $v_1 = [1 \;\; 2]^T$ and $v_2 = [1 \;\; -1]^T$. Find the new transformation matrix. *(You can leave the solutions as product of matrices form, without the need of actually computing the product)* 4

4. Get the coordinates of $p(t) = 2 + 3t$ with respect the basis $(p_1, p_2) = (t - 1, t + 1)$. Is this basis orthogonal with respect to the inner-product defined as

$$< p_1, p_2 > = \int_{-1}^{1} p_1(t) p_2(t) dt$$

3+2

5. A simple series circuit has three resistances r_1, r_2 and r_3, excited by a single voltage source V. Let $(v_1\ v_2\ v_3)$ be respectively the voltage drops across the resistances, (with obviously $(v_1 + v_2 + v_3) = V$). Show that $v = [v_1\ v_2\ v_3]$ lies in a vector space that is one-dimentinal ? 5

6. Imagine a linear transformation that reflects every vector on the $x - y$ plane (which serves as a mirror).

   (a) Give a matrix representation P of the above operator, with respect to the standard basis.

   (b) Verify that this matrix has the strange property that $P^2 = I$. What can be the physical significance of this relationship ?

3+2=5