Department of Electrical Engineering
ELL702, Nonlinear system,
Major Test, 2016-2017/I.
Max. time: 2 hours, Max. marks: 40.

Marks: Q1:6 , Q2:5 , Q3:5 , Q4:6, Q5:6, Q6:6, Q7: 6

- **Write clearly each step of your calculation.**

Q1. Suppose a nonlinear system is described by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\sin(x_1) - k(x_1 - x_3)$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = k(x_1 - x_3) + u$$
$$y = x_1$$

Find the range of k such that this system will be input-state feedback linearizable? What is the relative degree of this system?

Q2. Consider a system described by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -g(x_1) - h(x_1)\, x_2$$

(a) Find the condition on g(.) and h(.) such that origin is the isolated equilibrium point?

(b) Derive the condition on $h(x_1)$ and $g(x_1)$ such that the system will be asymptotically stable using the Lyapunov function

$$v(x_1, x_2) = \frac{1}{2} x_2^2 + \int_0^{x_1} g(x)dx.$$

Justify your answers by showing each step of your calculations.

Q3.(a) What are the disadvantages of limit cycles?

(b) Consider a system

$$\dot{x}_1 = \sigma x_1 + x_2$$
$$\dot{x}_2 = -x_1 + \sigma x_2$$

Classify the singular points corresponding to σ = -1, 0 and 1 respectively. Draw the respective phase plane trajectory.

Q4. Consider a system

$$\dot{x}_1 = x_2 + u$$
$$\dot{x}_2 = -u$$
$$y = x_1$$

(a) Determine the control law such that y(t) will track a specified trajectory $y_d(t)$ using input-output linearization.

(b) What is the internal dynamics of this system? Explain whether such controller will work in practice or not.

Q5. Consider a system and observer

System: $\dot{x}_1 = x_2$
$\dot{x}_2 = -2x_1 - x_2 + u$

Observer

$\dot{y}_1 = y_2 + L(x_1 - y_1)$
$\dot{y}_2 = -2y_1 - y_2 + u$

(a) Define a virtual system to synchronize the two systems justify your answer
(b) Find the restriction on gain L using contraction analysis to ensure the synchornisation.

Q6. Consider a periodic system

$\dot{x}(t) = A(t)x(t)$, $x(0)$ is the initial condition. x = n×1 vector

(a) Define the fundamental matrix **X** and the state transition matrix **Φ** of the system.
(b) Suppose $z = P(t)x$ and **Z** is the fundamental matrix of $\dot{z}=\bar{A}z$. Express P(t) in terms of **X** and **Z**.

Q7. Suppose an LTI system $\dot{x} = Ax,\ x(0) = x_0$, is shown to be asymptotically stable using a Lyapunov function $V(x) = x^T P x > 0$ where $\dot{V}(x) = x^T Q x < 0$.

(a) Show that the norm of x(t) satisfies the following relation:

$$\|x(t)\| \le A e^{-\alpha t}$$

Also express α in terms of P and Q.

(b) Show that $P = \int_0^\infty e^{A^T t} Q e^{At}\, dt$ where $Q=HH^T>0$.

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