

Marks: Q1: 7, Q2: 7, Q3: 6

> Write clearly each step of your calculation.

Q1. Consider a system described by

$$\ddot{\theta} + \dot{\theta} + \sin \theta = 0$$

Verify the stability of the equilibrium point using two different Lyapunov function:

(a)  $V(\theta, \dot{\theta}) = (1 - \cos \theta) + \frac{\dot{\theta}^2}{2}$

(b)  $V(\theta, \dot{\theta}) = \frac{1}{2} \dot{\theta}^2 + \frac{1}{2} (\dot{\theta} + \theta)^2 + 2(1 - \cos \theta)$

Q2. @ Consider a function

$$f(x) = \begin{bmatrix} x_2 \\ -\text{sat}(x_1 + x_2) \end{bmatrix}$$

Show that  $f(x)$  is Lipschitz continuous? Find a Lipschitz Constant?

(b) What do you mean by "finite escape time"? Explain it briefly through an example.

Q3. @ Consider a scalar system described by

$$\dot{x} = -x^3$$

Using linearization method verify the stability of the <sup>equilibrium</sup> point. Also verify whether the equilibrium point is ~~asymptotically~~ asymptotically stable using  $V(x) = \frac{1}{2} x^2$ .

(b) Consider a system transfer function  $h(p) = \frac{N(p)}{D(p)}$ . verify whether the following systems are SPR or not:

(i)  $h_1(p)$  where  $N(p) = p-1$ , &  $D(p) = p^2 + p + 1$

(ii)  $h_2(p)$  where  $N(p) = p+1$  &  $D(p) = p^2 - p + 1$ .