6/5/'17 ELL703 OPTIMAL CONTROL

MAJOR TEST — Marks: 40, Time: 2 hrs.

Q.1 (a) Let (x^*, u^*) be optimal solution for minimization of PI $L(x,u)$ subject to constraint $f(x,u)=0$. Now, the constrained is changed to $f=df$. We are required to remain ~~to remain~~ at optimal solution (x^*+dx, u^*+du). Derive expressions for dx and du in terms of df.

(b) Find the optimal curve $x^*(t)$ which minimizes
$$J = \int_0^{\pi} (\dot{x}(t))^2 \, dt \text{ with } x(0)=0 \text{ and } x(\pi)=1.$$

Q.2:- Consider finite time LQ Regulator problem where plant is $\dot{x}(t) = A(t)x(t) + B(t)u(t)$ with cost functional
$$J(x(t_0),t_0) = \frac{1}{2}x'(t_f)F(t_f)x(t_f) + \frac{1}{2}\int_{t_0}^{t_f}(x'(t)Q(t)x(t) + u^T(t)R(t)u(t))\,dt.$$

(a) Show whether optimal value of PI is given by
$$J^*(x^*(t),t) = \frac{1}{2}x^{*\prime}(t)P(t)x^*(t) \text{ or not.}$$
where $P(t)$ is solution of Riccati eq^n

(b) Show whether $P(t)$ is Positive definite for all $t \in [t_0, t_f)$ or not. Assume $F \geq 0$, $Q \geq 0$ & $R > 0$

Q.3:- Consider a plant $x_{k+1} = x_k + 2u_k$, x_0 is given, with PI $J_0 = S_N(x_N - r_N)^2 + \frac{r}{2}\sum_{k=0}^{N-1} u_k^2$.

(i) Suppose final state x_N is fixed at $x_N = r_N$. Find the optimal control sequence u_k^*, optimal state trajectory x_k^* which minimizes PI and the optimal PI.

(ii) If the final state is free but final time N is fixed, then find optimal control sequence u_k^*, optimal state trajectory $x^*(k)$ and optimal PI.

(iii) What is the effect of $r \to 0$ in part (ii)

Q.4: (a) Derive the Hamilton-Jacobi-Bellman equation when plant $\dot{x}(t) = f(x(t), u(t), t)$ and $J(x(t_0),t_0) = \int_{t_0}^{t_f} V(x(t),u(t),t)\,dt + S(x(t_f),t_f)$

~~(b) Using H-J-B~~

Q.4(b):- Using HJB framework, find the closed loop control for system $\dot{x}(t) = -2x(t) + u(t)$

with PI = $J = \int_0^\infty \left(x^2(t) + u^2(t)\right) dt$

Hint: use $J^* = p\,x^2(t)$ where p is constant.

Q.5:- Consider plant ~~$\dot{x}(t) = x - u(t)$~~, $\dot{x}(t) = x(t) - u(t)$.

It is desired to find control $u(t)$ which drives the system with any initial condition $x(0)$ to zero in ~~minit~~ minimum time. The input is constrained and $|u(t)| \leq 1$. Formulate the problem and provide complete solution using Pontryagin Principle. ~~Find~~ ~~(i) solution~~ ~~optimal solution $u^*(t)$ and $x^*(t)$~~

(i) Find solution for costate $\lambda(t)$ and $u^*(t)$ in terms of $\lambda(T)$ for all possible cases.

(ii) Solve state eq^n for all possible values of input $u^*(t)$ if $x(T) = 0$.

(iii) Sketch switching curve and sample trajectories in phase plane

(iv) Find the optimal feedback control and optimal cost in terms of $x(0)$

(v) In terms of $x(0)$, when does this optimal control problem have a solution.