2/2/'17 **ELL703 OPTIMAL CONTROL**

Minor Test I Marks: 20, Time: 1 hr.

Q.1:- Consider extremization of

$$L(x,u) = \frac{1}{2}[x \; u]\begin{bmatrix}2 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}x\\ u\end{bmatrix} + [1 \; 0]\begin{bmatrix}x\\ u\end{bmatrix}$$

subject to constraint $f(x,u) = x - u + 3 = 0$. (i) Find the stationary point $(x^*, u^*) = (x,u)^*$. (ii) Verify $L(x,u)^*$ is minimum or not. (iii) Find the value of λ constraint optima and compute gradients of $L(x,u)$ and $f(x,u)$ at constrained optima. (7)

Q.2:- A mechanical system is ~~desc~~ described by $\ddot{x}(t) = u(t)$. Find the optimal control and states by minimizing $J = \frac{1}{2}\int_0^5 u^2(t)\,dt$ satisfying the boundary conditions $x(0)=2, \dot{x}(0)=2, x(5)=0, \dot{x}(5)=0$ (7)

Q.3:- Consider ~~of~~ extremization of PI functional

$$J(x,t) = \int_{t_0}^{t_f} V(x, \dot{x}, t)\,dt$$

subject to plant condition $g(x,\dot{x},t)=0$ with fixed end point conditions $x(t_0)=x_0$, $x(t_f)=x_f$. Derive the set of necessary conditions for extremum. (6)

[redacted]

$= \frac{1}{2}[x \; u]\begin{bmatrix}2x\\ 2x+u\end{bmatrix} + x$

$= \frac{1}{2}[2x^2+2xu+u^2] + x$

$L = x^2 + xu + u^2/2 + x$

$F = x - u + 3$

$H = L + \lambda F$

$H = x^2 + xu + u^2/2 + x + \lambda(x-u+3)$

$H_x = 2x + u + 1 + \lambda = 0$

$H_u = x + u \cancel{+} \lambda = 0$

$H_\lambda = x - u + 3 = 0$

$x = u - 3$

$u - 3 + u - \lambda = 0$

$2u - 3 = \lambda$

$2u - 6 + u + 1 + 2u - 3 = 0$

$5u = 8$

$u = 1.6$

$x = -1.4$

$\frac{8}{5}$ $\frac{13}{10}$