ELL 705 MAJOR TEST

Duration: 2 hours

Total Marks: 35

Instructions

1. Show all relevant steps clearly and briefly.
2. If needed, make suitable assumptions. But, state them clearly.

Question 1. *(10 marks)* Consider the process $x_{k+1} = ax_k + v_k, y_k = 0.5x_k + w_k$, where x_k is the state, y_k is the output, a is an unknown parameter and v_k, w_k are independent zero-mean Gaussian white-noise processes with variances σ^2. Design an Extended Kalman Filter (predicted state) to estimate parameter a from output measurements.
[*Note:-* For the state-space model,

$$\begin{aligned} x_{k+1} &= \Phi x_k + v_k, \\ y_k &= Cx_k + w_k, \end{aligned}$$

where v_k and w_k are assumed to be independent zero-mean white-noise processes with covariances Σ_v and Σ_w, respectively, the Kalman Filter (predicted state) takes the form,

$$\begin{aligned} \hat{x}_{k+1|k} &= \Phi\hat{x}_{k|k-1} - K_k(\hat{y}_k - y_k), \\ \hat{y}_k &= C\hat{x}_{k|k-1}, \\ K_k &= \Phi P_k C^T(\Sigma_w + CP_kC^T)^{-1}, \\ P_{k+1} &= \Phi P_k \Phi^T + \Sigma_v - \Phi P_k C^T(\Sigma_w + CP_kC^T)^{-1}CP_k\Phi^T.] \end{aligned}$$

Question 2. *(18 marks = 5 + 6 + 7)* Consider the estimation of the ("almost" constant) temperature θ from noisy measurements $y_k = \theta + w_k$, where y_k is the measured temperature and w_k is a zero-mean Gaussian white-noise processes with variance R.
a) Using the method of Recursive Least Squares, show that $\hat{\theta}_{k+1} = \hat{\theta}_k + K_{RLS}(y_{k+1} - \hat{\theta}_k)$. Find K_{RLS}.
b) The dynamics of actual temperature can be modelled as $x_{k+1} = x_k + v_k$, where v_k is a zero-mean Gaussian white-noise process with variance Q and independent to w_k. The measurement equation remains $y_k = x_k + w_k$ as above. Suppose a Kalman Filter (predicted state) is to be designed to estimate the actual temperature from measurements.
i) Show that the covariance propagation equation is $P_{k+1} = P_kQ/(P_k + Q) + R$.
ii) Assuming that P_k converges to a constant value P, find P?
iii) For what condition is $P < Q$?
iv) Does P_k converge to P? (*Hint:-* Study fixed points and stabilities of the map $P_k \to P_{k+1}$).
c) Construct the Kalman Filter in the the predicted state framework ($\hat{x}_{k+1|k}$) and in the filtered state framework ($\hat{x}_{k+1|k+1}$). Compare the two Kalman gains obtained in these frameworks?

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Question 3. *(7 marks)* The discrete-time, Extended Kalman Filter is to be used to estimate the parameters $1/T$ and ν in a macroscopic highway traffic model.

A dynamical equation is given by,

$$u_j^{n+1} = u_j^n + \Delta t\Big\{ - u_j^n \frac{u_j^n - u_{j-1}^n}{\Delta x} - \frac{1}{T}\Big[u_j^n - a - b\rho_j^n + \frac{\nu(\rho_{j+1}^n - \rho_j^n)}{\rho_j^n \Delta x}\Big]\Big\} + w_j^n,$$
$$z_j^n = u_j^n + v_j^n,$$

where,
n indexes time,
j indexes a section of the road,
Δx is the section length of the road,
ρ_j^n is the vehicles per unit length in the section between x_j and x_{j+1} at time $n\Delta t$,
u_j^n is the average of speeds of vehicles in the section between x_j and x_{j+1} at time $n\Delta t$,
T is the reaction time,
Δt is the time interval,
a and b are parameters that are known,
ν is sensitivity factor,
z_j^n is the measured value at time n,
w_j^n and v_j^n are zero-mean white Gaussian noise processes with covariances Q and R, respectively.

Redefine the parameters $1/T$ and ν as states and obtain the (nonlinear) dynamical equations that can be used for the design.

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