ELL 705 MINOR TEST 2

Duration: 1 hour

Total Marks: 300

Instructions
1. Show all relevant steps clearly and briefly.
2. If needed, make suitable assumptions. But, state them clearly.

Question 1. *(150 marks = 75 + 75)* Consider the noisy measurement of an unknown parameter θ, $y(t) = \theta + w(t)$, where $w(t)$ is a zero mean white noise with variance σ^2. Data is collected at times $t = 1, 2, ...N$ and is denoted as $y(1), y(2), ..., y(N)$.

a) Suppose $\theta = \theta_0$, a constant. Let $\hat{\theta}$ denote its least-squares parameter estimate. Show that $E\{(\hat{\theta} - \theta_0)^2\} = 1/N$.

b) Suppose now that the parameter $\theta = \theta(t)$ varies with time. Using recursive-least squares with a forgetting factor λ, $0 < \lambda < 1$, show that the recursion relation is of the form $\hat{\theta}_{N+1} = \hat{\theta}_N + K_N(y(N+1) - \hat{\theta}_N)$. What is K_N?

Question 2.*(150 marks = 100 + 50)*

a) Show that $E\{(\frac{\partial L}{\partial \theta})^2\} = -E\{\frac{\partial^2 L}{\partial \theta^2}\}$, where $L = \log p(y|\theta)$ is the log-likelihood function, θ is the parameter and y is the output. Recall that this expression is the Fisher Information $I(\theta)$.

b) Suppose that we have a single observation of a random process following a Gaussian distribution, $Y \sim N(\mu, \sigma^2)$. Assume that we are interested in knowing $\theta = \mu$ when σ^2 is given. Calculate the Fisher Information $I(\mu)$.

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