(2 1/2)

Sekhar

ELL720: Advanced Digital Signal Processing (Spring 202

Minor # 1

Name:

Time allowed: 60 mins

Entry No

Answer in the space provided. Justify your answer clearly. Answers without justification will receive no credit.

1. Consider the two systems shown below: (8 points)

System A: $x[n] \rightarrow \boxed{\downarrow M}_{2} \xrightarrow{w_A[n]} \boxed{\uparrow L}_{3} \xrightarrow{} y_A[n]$ System B: $x[n] \rightarrow \boxed{\uparrow L}_{3} \xrightarrow{w_B[n]} \boxed{\downarrow M}_{2} \xrightarrow{} y_B[n]$

(a) For $M = 2, L = 3$, and any input $x[n]$, will $y_A[n] = y_B[n]$? Justify your answer.

(b) If $M = L$, are the two systems identical? Justify your answer.

1) ~~Lets Consider system A~~
to prove that $y_A[n] = y_B[n]$
if the output match then we are done.

?

to The system A
as we down sample by factor of 2, alternative samples gets dropped
so and we have no Low pass filter in between because in second system B
after upsampling signal can be bandlimited, if we upsample then
we will not be able to recover the signal.

1

System A

$$w_a[n] = \frac{1}{2}\sum_{i=0}^{1} X\left(e^{j\left(\frac{\omega-2\pi i}{2}\right)}\right)$$

Then we upsample by factor of $L=3$

Then The final $y_A[n] = W_A(e^{\omega jL}) = W_A(e^{3\omega j})$

$$= \sum w_a[n] \cdot e^{-jk\omega \cdot 3}$$

Substitute value of $w_a[n]$

$$= \sum_{k=-\infty}^{\infty} \left\{\frac{1}{2}\sum_{i=0}^{1} X\left(e^{j\left(\frac{\omega-2\pi i}{2}\right)}\right)\right\} e^{-jk\omega \cdot 3}$$

System B

$$w_B = X(e^{j\omega 3})$$

$$w_B = \sum_{k=-\infty}^{\infty} x[n] \cdot e^{-jk\omega \cdot 3}$$

Now down sample by factor of $M=2$

$$y_B[n] = \frac{1}{2}\sum_{0}^{1} w_B\left(e^{j\left(\frac{\omega-2\pi i}{2}\right)}\right)$$

Substitute value of w_B

$$y_B = \frac{1}{2}\sum_{i=0}^{1}\left\{\sum_{k=-\infty}^{\infty} x[n] e^{-3jk\omega}\right\} e$$

?

0

0

0

b) If $M=L$ then the signal will not be same as we dont know
that the after upsampling the signal values are bandlimited or not
so we can say 2 system are not identical.
It can become identical if we have the low pass filter which makes the
signal band limited.

2. Consider the system: $y[n] = x[n] - 0.95x[n-6]$. (12 poi.

(a) Sketch its pole-zero pattern with proper labelling.

(b) Using the pole-zero plot, sketch its magnitude response with proper labelling.

(c) If the sampling frequency is 1 kHz, determine the frequencies of the analog sinusoids that will be strongly attenuated by this filter.

As we have difference eq

$y[n] = x[n] - 0.95\, x[n-6]$

it implies that present output depends on the present input and the past input delayed by 6

Apply Z Transform on the difference equation.

$Y(Z) = X[Z] - 0.95 \cdot Z^{-6} \cdot X[Z]$

$\Rightarrow Y(Z) = X(Z) - 0.95 \cdot Z^{-6} \cdot X[Z]$

$\Rightarrow Y(Z) = X[Z] \cdot (1 - 0.95 \cdot Z^{-6})$

so to obtain the transfer function / system function $H(Z)$

$H(Z) = \frac{Y(Z)}{X(Z)} = (1 - 0.95 \cdot Z^{-6})$ or $= \frac{Z^6 - 0.95}{Z^6}$

$(0.95)^{1/6} e^{j\frac{2\pi k}{6}}$

so to obtain the poles and zero's

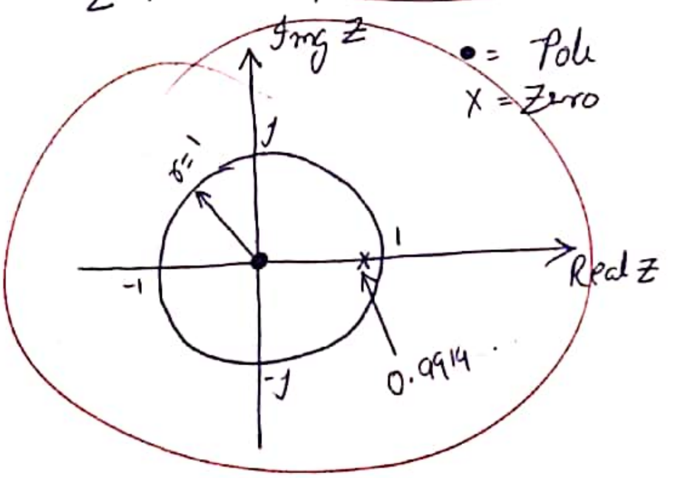
$Z^6 - 0.95 = 0$

$Z^6 = 0.95$

$Z = \sqrt[6]{0.95} = 0.99148$ (Zero)//

as pole will be at $Z = 0$ as denominator approach 0 if

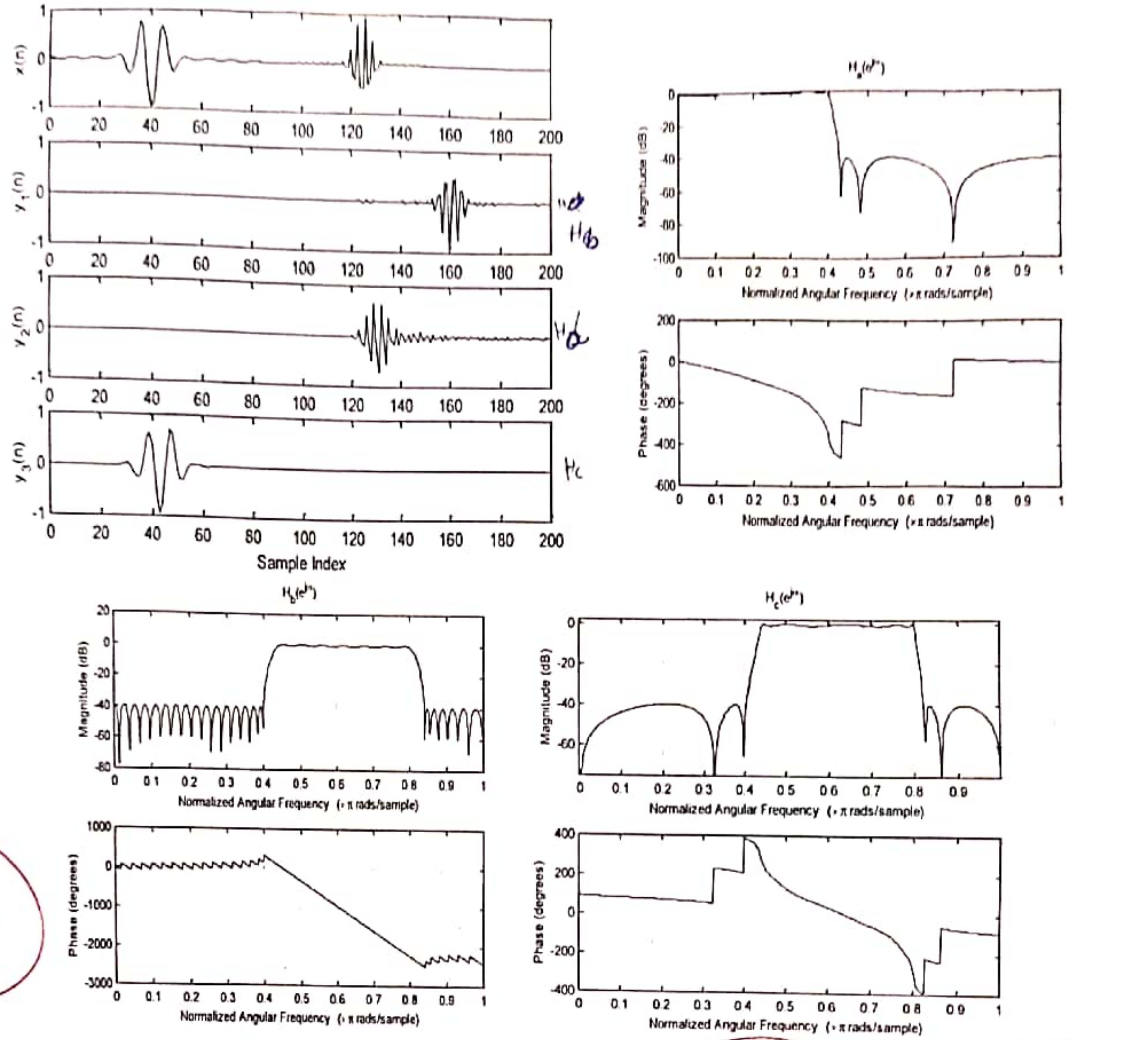
$Z \to 0$. so pole is at 0 (origin)//



$1\frac{1}{2}$

~~Sharp dips~~

3. In the figure below, $x[n]$ is filtered separately by three different digital filters $H_a(e^{j\omega})$, $H_b(e^{j\omega})$, and $H_c(e^{j\omega})$, to produce three output sequences which are shown. The frequency responses of the three digital filters are also shown. For each of the output sequences, determine the filter that produced it. Justify your answer. (10 points)



0

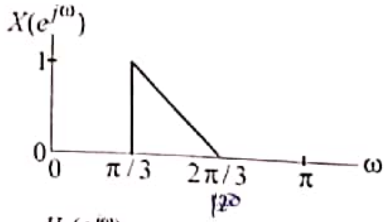
for sig 1 ~~x(n)~~ $y_1(n)$ we can see that upto 100 HZ we we have made the response as 0 and then this is certain amplitude (~~signal got reversed~~) so reversing of signal can be possible by $H_b(e^{j\omega})$ & $H_c(e^{j\omega})$ and magnitude gets Zero which means dips of signal $H_b(e^{j\omega})$ (magnitude) those dips makes signal Zero. so $y_1(n]$ ~~but~~ is done by $H_b(e^{j\omega})$

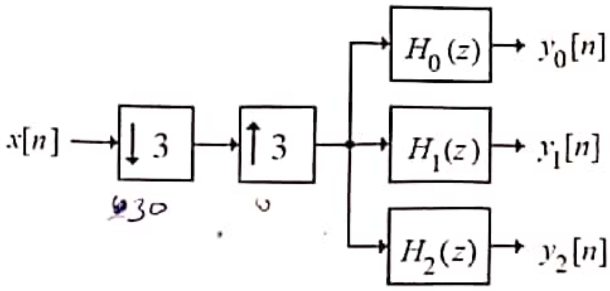
Now $y_2(n)$ as their are certain disturbance in signal $y_2(n)$ which can be done by $H_a(e^{j\omega})$ as the amplitude of $y_2(n)$ gets Zero in first 100 frequencies which is done by magnitude effect of filter $H_a(e^{j\omega})$

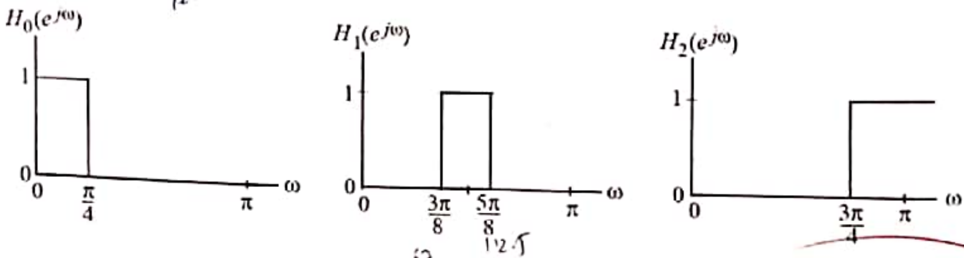
$y_3(n)$ will be obtained by filter $H_c(e^{j\omega})$.

Justification is incorrect.

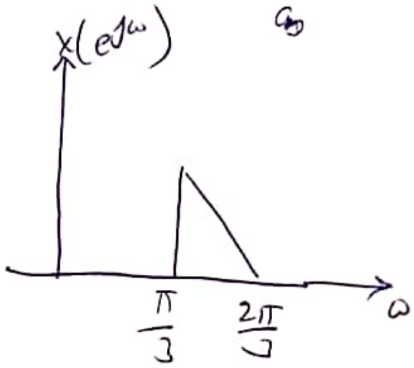
4. Consider the multirate structure shown below. The frequency responses of the systems $H_i(z), i = 0, 1, 2$ (with real impulse responses) are also shown. If the input $x[n]$ is a real sequence with DTFT as shown, sketch the DTFTs of the outputs $y_0[n]$, $y_1[n]$ and $y_2[n]$. Show all intermediate steps for credit. (20 points)





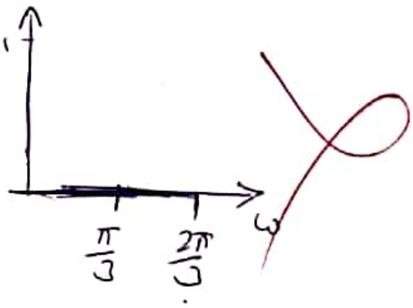


~~$y_0[n]$~~

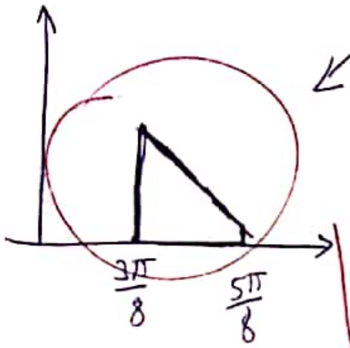


as after down sampling & upsampling frequency have no effect on it (change) so when $H_0(e^{j\omega})$ is convolved with the signal it will make the response zero

$y_0[n] =$



$y_1[n] =$



(a trapezoidal wave will form) as filter response b/w $3\pi/8$ to $5\pi/8$ which covers some of the value of signal as filter response is 1 (constant) it will retain the signal value.

N