EPL202: MinorII: IIT DELHI

Max marks 20 Time 1 hrs.

March 24, 2014

1. Consider a particle of mass m and charge q executing simple harmonic motion in one dimension along the x-axis with frequency ω. You can take $q > 0$. An electric field $\boldsymbol{E} = E\hat{x}$ is applied on this partilce, where $E > 0$ and $\hat{x}$ is the unit vector along x-axis. Find out the energy spectrum and the ground state wave function. Calculate the average position of the particle in the ground state and show how it depends on E. 6

2. Consider an electron in the eigenstate of the $\hat{S}_z$ operator with eigenvalue $\frac{1}{2}\hbar$. One makes a measurement of the Spin operator $\hat{S}_n = \boldsymbol{S}\cdot\boldsymbol{n}$, where $\boldsymbol{n}$ is an unit vector in the $z-x$ plane which makes an angle $\frac{\pi}{3}$ with the positive z-axis. Find out the probability of of obtaining spin eigenvalue $-\frac{\hbar}{2}$. Explain the outcome. 4

3. We know, spherical harmonics($Y_{l,m}(\theta,\phi)$ or $|l,m\rangle$) are simultaneous eigenfunction of the operator L^2 and L_z, and, $L^2Y_{l,m} = l(l+1)\hbar^2 Y_{l,m}$ and $L_zY_{l,m} = m\hbar Y_{l,m}$ Now consider three such functions, namely, $Y_{1,1}, Y_{1,0}$ and $Y_{1,-1}$. Show that the simultaneous eigenfunctions of L^2 and L_x are given by

$$\begin{aligned} \psi_1 &= \frac{1}{2}(Y_{1,1} + Y_{1,-1} + \sqrt{2}Y_{1,0}) \\ \psi_2 &= \frac{1}{\sqrt{2}}(Y_{1,1} - Y_{1,-1}) \\ \psi_3 &= \frac{1}{2}(Y_{1,1} + Y_{1,-1} - \sqrt{2}Y_{1,0}) \end{aligned}$$ 5

4. A hydrogen like atom has one electron in the outer shell and and has a nucleus of positive charge Ze and properties (eigenfunctions and eigenvalues) are similar to hydrogen atom. A wave function for such a hydrogen like atom is

$$\psi(r,\theta,\phi) = \frac{1}{81}\sqrt{\frac{2}{\pi}}Z^{3/2}(6 - Z\frac{r}{a_0})Z\frac{r}{a_0}\exp(-Z\frac{r}{3a_0})\cos\theta$$

(a) Find out the corresponding values of the quantum number n, l, m.
(b) Construct from the above $\psi(r,\theta,\phi)$ another wave function with same

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value of n, l, but with a different magnetic quantum number $m + 1$.

(c) Calculate the most probabble value of r for an electron in the state corresponding to ψ given in the question. $2 + 1 + 2$

You may like to use $L_x + iL_y = \exp(-\phi)\frac{\partial}{\partial\theta} + i\exp(i\phi)\cot\theta\frac{\partial}{\partial\phi}$. And the hydrogen atom with principal quantum number n decays at large r as $exp(-\frac{r}{na_0})$.