Indian Institute of Technology, Delhi
Department of Physics
EPL208 Electrodynamics and Plasmas
Second Semester 2013-2014

Major
Duration: 2 hour

Marks: 40
Date: 05 May 2014

1. (a) Suppose $V = 0$ and $\boldsymbol{A} = A_0 \sin(kx - \omega t)\hat{y}$, where A_0, ω and k are constants. Find $\boldsymbol{E}$ and $\boldsymbol{B}$, and check that they satisfy Maxwell's equations in vacuum. What condition must you impose on ω and k?

   (b) A particle of charge q moves in a circle of radius a at constant angular velocity ω. Assume that the circle lies in the xy plane, centered at the origin, and at time $t = 0$ the charge is at $(a, 0)$, on the positive x axis. Find the Lienard-Wiechert potentials for points on the z axis.

(4+4)

. (i) In the case of oblique incidence of em wave on a boundary between two linear media, show that the angle of reflection is equal to angle of incidence and the angle of refraction follows the well known Snell's law.

(ii) Consider a Hertzian dipole an infinitesimal current element $I.dl$ located at the origin of co-ordinate system such that dl is oriented in z direction and carries a current $I(t)= I_0 \cos \omega t$. Calculate the retarded potential $\boldsymbol{A}$ and then the $\boldsymbol{B}$ and $\boldsymbol{E}$ fields. Further show that the power radiated by the dipole is given by

$$P_{rad} = 40\pi^2 \left[\frac{dl}{\lambda}\right]^2 I_0^2$$

(4+6)

(α) Let us consider propagation of electromagnetic wave in plasma with magnetic field $\boldsymbol{B_0} = B_0\,\hat{z}$. We also consider perpendicular propagation $\boldsymbol{k} \perp \boldsymbol{B_0}$ (say $\boldsymbol{k} = k\,\hat{x}$) and transverse waves ($\boldsymbol{k} \perp \boldsymbol{E_1}$ however $\boldsymbol{E_1} \parallel \boldsymbol{B_0}$). The plasma has a density n_0 and electron temperature T_e. Calculate the dispersion relation for this electromagnetic wave.

(β) The dispersion relation for electromagnetic waves parallel to $\boldsymbol{B_0}$ is given by

$$\omega^2 - c^2k^2 = \frac{\omega_p^2}{1 \mp (\omega_c/\omega)}$$

where − sign is for R wave and + for L wave. Calculate the cutoffs and resonances for these waves.

(7+4)

4. (i) A plasma with an isotropic velocity distribution is placed in a magnetic mirror trap with mirror ratio $R_m = 4$. There are no collisions, so the particles in the loss cone simply escape, and the rest remain trapped. What fraction is trapped?

(ii) Consider a rectangular wave guide with dimensions 2.28 cm x 1.01 cm. What TE modes will propagate in this wave guide, if the driving frequency is 1.70×10^{10} Hz? Suppose you wanted to excite only *one* TE mode; what range of frequencies could you use? What are the corresponding wavelengths (in open space)?

(iii) Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω and phase angle zero that is (a) traveling in the negative x direction and polarized in the z direction; (b) traveling in the direction from the origin to the point (1, 1, 1), with polarization parallel to the xz plane.

(3+4+4)

Constants:

$e = 1.60 \times 10^{-19}$ C $m_e = 9.11 \times 10^{-31}$ Kg $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ H/m $c = 3.0 \times 10^{8}$ m/s

Lienard-Wiechert potentials for a moving point charge:

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{qc}{(\mathscr{r}c - \boldsymbol{\mathscr{r}} \cdot \mathbf{v})}, \qquad \mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \frac{qc\mathbf{v}}{(\mathscr{r}c - \boldsymbol{\mathscr{r}} \cdot \mathbf{v})}$$

Fields of a moving point charge:

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\mathscr{r}}{(\boldsymbol{\mathscr{r}} \cdot \mathbf{u})^3} [(c^2 - v^2)\mathbf{u} + \boldsymbol{\mathscr{r}} \times (\mathbf{u} \times \mathbf{a})]. \quad \mathbf{B}(\mathbf{r}, t) = \frac{1}{c} \hat{\boldsymbol{\mathscr{r}}} \times \mathbf{E}(\mathbf{r}, t).$$

where the symbols have there usual meanings

$$\int_0^{\pi} \sin^3\theta \, d\theta = \frac{4}{3}$$
