Physics Department

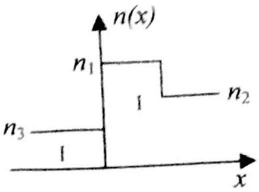
Fiber and Integrated Optics (EPL442/PYL413): Major

November 20, 2016

Maximum marks: 40

ALL QUESTIONS ARE COMPULSORY

1. Give brief answers to the following: **(8x2=16)**



a) Consider a planar waveguide with a refractive index profile shown in the figure. State the range of effective index values for guided modes. In which cladding region will the modal field penetrate farther? Give reasons for your answer

b) A 100 ps Gaussian optical pulse centered at 1300 nm gets broadened to 200 ps after propagating through a certain length of an optical fiber. What is the spectral width $\Delta\lambda_0$ of the output pulse?

c) For pure silica, $n(\lambda_0) = 1.451 - 0.003\left(\lambda_0^2 - \frac{1}{\lambda_0^2}\right)$ where λ_0 is measured in μm. How much time will it take for the peak of an input Gaussian pulse to exit after travelling through a length of 1 km of this medium? Assume a wavelength of 800 nm.

d) The ends of two step index single mode fibers having V values of 2 and 1.8 are perfectly aligned with each other. What should be the ratio of the core radii of the two fibers so that the joint is lossless. Neglect reflection loss.

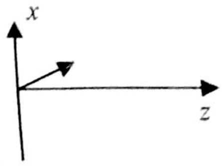
e) Consider a fiber optic link of 100 km length with a single mode fiber having a loss coefficient of 0.3 dB/km. The signal power input is 10 mW. Calculate the signal power in mW received by the receiver.

f) Consider a fiber with $a = 4$ μm $n_1 = 1.451$ and $n_2 = 1.447$. At which of the following two wavelengths will the measured attenuation coefficient of the fiber depend on input excitation conditions: (a) 1300 nm and (b) 500 nm. Give reasons for your answer.

g) The output beam from the end of a single mode fiber has a divergence of 4.74°. Obtain the Gaussian spot size of the fundamental mode of the fiber. $\lambda = 1300$ nm

h) Consider a fiber optic communication system operating at 2.5 giga bits per second. What is the bandwidth requirement under NRZ and RZ schemes?

2. Consider a medium with



$$n^2(x) = 2.25 - 0.01\,x; \qquad x > 0$$
$$= 2.25 \qquad x < 0$$

A ray is launched from the point $x = 0$, $z = 0$ making an angle of 30° with the z-axis.

a) Calculate the value of x where the ray will turn back.

b) Obtain an equation for the ray path in this medium.

c) At what value of z will the ray re-intersect the z-axis? **(6)**

3. Consider a Gaussian pulse given by

$$\psi(z = 0, t) = Ae^{-t^2/\tau_0^2}e^{i\omega_0 t}$$

propagating through free space.

a) Obtain the frequency spectrum of the pulse.

b) Show that the pulse will propagate undistorted.

c) Obtain the group velocity of the pulse. **(6)**

4. Consider a step index optical fiber with $n_1 = 1.450$, $n_2 = 1.447$ and $a = 5$ μm.
   a) Obtain the cut off V-values of LP_{01}, LP_{11}, LP_{02}, LP_{21} and LP_{12} modes.
   b) For what wavelength range will the fiber support only the LP_{01} and LP_{11} modes?
   c) If a fiber supporting LP_{01}, LP_{11}, LP_{02} and LP_{21} modes is excited by the following incident field at a wavelength of 800 nm,

$$E_{in} = A\, e^{-r^2/w_0^2}$$

(where A and w_0 are arbitrary constants) which specific modes in the fiber will get excited? Give reasons for your answer. **(6)**

5. Consider a single mode fiber with $n_1 = 1.458$, $n_2 = 1.447$ and $a = 2.3$ μm operating at 1500 nm.
   a) Obtain the dispersion coefficient of the fiber at this wavelength in units of ps/km-nm.
   b) At this wavelength is the dispersion normal or anomalous?
   c) What should be the dispersion coefficient of a dispersion compensating fiber of length 20 km required to compensate the dispersion of 100 km of this fiber?

Refractive index of silica: $n(\lambda_0) = 1.451 - 0.003\left(\lambda_0^2 - \frac{1}{\lambda_0^2}\right)$; λ_0 in micrometers. **(6)**

Some useful formulas:

- $\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2}\frac{dn^2(x)}{dx}$
- Zeroes of Bessel functions:
  $J_0(x) = 0$ at $x = 2.405, 5.520, 8.654, 11.79$; $J_1(x) = 0$ at $x = 0, 3.832, 7.016, 10.174$
  $J_2(x) = 0$ at $x = 0, 5.136, 8.417, 11.620$
- Eigenvalue equations for a step index fiber:
  $U\frac{J_1(U)}{J_0(U)} = W\frac{K_1(W)}{K_0(W)}$; $l = 0$; $U\frac{J_{l-1}(U)}{J_l(U)} = -W\frac{K_{l-1}(W)}{K_l(W)}$; $l = 1, 2, 3 \ldots$
- $\frac{w}{a} = 0.65 + \frac{1.619}{V^{1.5}} + \frac{2.879}{V^6}$
- $b(V) = \left(1.1428 - \frac{0.996}{V}\right)^2$
- $\int_{-\infty}^{+\infty} e^{-px^2+qx}dx = \sqrt{\frac{\pi}{p}}e^{q^2/4p}$
- $\tau^2(z) = \tau_0^2\left(1 + \frac{4\alpha^2z^2}{\tau_0^4}\right)$; with $\alpha = -\frac{\lambda_0^2}{2\pi c}D$
- Group index: $N(\lambda_0) = n(\lambda_0) - \lambda_0\frac{dn}{d\lambda_0}$
- Refractive index of silica: $n(\lambda_0) = 1.451 - 0.003\left(\lambda_0^2 - \frac{1}{\lambda_0^2}\right)$
- Material dispersion: $\Delta\tau_{Mat} = -\frac{\lambda_0}{c}\frac{d^2n}{d\lambda_0^2}$
- Approximate formula for waveguide dispersion in step index fibers ($\Delta n = n_1 - n_2$):
  $\Delta\tau_{WG} = -\frac{\Delta n}{c\lambda_0}V\frac{d^2(bV)}{dV^2}$; $V\frac{d^2(bV)}{dV^2} \approx 0.080 + 0.549(2.834 - V)^2$
- Gaussian approximation for single mode fibers:
  $\psi(r) = Ae^{-r^2/w^2}$; $w = a\left(0.65 + \frac{1.619}{V^{1.5}} + \frac{2.879}{V^6}\right)$