

✓ 1. (a) Prove the convolution theorem:

$$\mathcal{F}\left\{\int_{-\infty}^{\infty} dx' g(x') h(x - x')\right\} = G(f_x) H(f_x).$$

In the above equation,  $G$  and  $H$  denote the Fourier transforms of  $g$  and  $h$  respectively. (5 points)

(b) Plot the function:  $p(x) = \mathcal{F}^{-1}\{\text{sinc}^2(f_x)\}$ . Here  $\text{sinc}(u) = \frac{\sin(\pi u)}{\pi u}$ . (5 points)

2. The Wigner distribution function is defined as: *conjugate*

$$W(x, f_x) = \int_{-\infty}^{\infty} d\xi g(\xi + x/2) g^*(\xi - x/2) \exp(-i2\pi f_x \xi).$$

(a) Evaluate  $\int_{-\infty}^{\infty} dx W(x, f_x)$ . (5 points)

$$g^2(\xi + x/2)$$

(b) Evaluate  $\int_{-\infty}^{\infty} df_x W(x, f_x)$ . (5 points)

3. (a) State TRUE or FALSE with reasoning: For signals  $g(x)$  bandlimited to frequencies  $f_x : (-B, B)$

$$\int_{-\infty}^{\infty} dx' g(x') \text{sinc}[2B(x - x')] = \frac{1}{2B} g(x).$$

(5 points)

✓ (b) IIT Delhi central library needs a barcode scanner for reading barcodes on the books. A typical barcode is shown in the figure below. It has a width of 1 inch (2.54

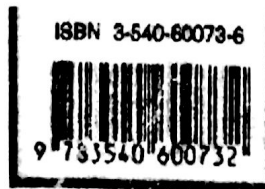


Figure 1: Typical barcode on a book. Barcode width = 1 inch.

cm) and the thinnest vertical bar is approximately 0.5 mm wide. The scanner device has a lens which images the barcode onto a linear array (single row) of detector pixels with suitable de-magnification. Estimate the minimum number of pixels needed in the linear detector array for proper functioning of the barcode scanner. (5 points)

*Mr.*

4. Consider the relation:

$$U(P_0) = \frac{1}{4\pi} \int \int_{S_1} ds \left( \frac{\partial U}{\partial n} G - U \frac{\partial G}{\partial n} \right).$$

Here  $S_1$  is the surface  $z = 0$  and  $U, G$  are scalar fields satisfying the Helmholtz equation. Simplify the above relation with appropriate reasoning when

(a)

$$G = G_- = \frac{\exp(ikr_{01})}{r_{01}} - \frac{\exp(ik\tilde{r}_{01})}{\tilde{r}_{01}}$$

(5 points)

(b)

$$G = G_+ = \frac{\exp(ikr_{01})}{r_{01}} + \frac{\exp(ik\tilde{r}_{01})}{\tilde{r}_{01}}$$

(5 points)

In the above equations,  $r_{01}$  and  $\tilde{r}_{01}$  denote distances between a point in the aperture (in the  $z = 0$  plane) to two image points  $P_1$  and  $\tilde{P}_1$  respectively. (You may leave your answer in terms of the directional derivative.)

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