Indian Institute of Technology, Delhi

Minor 1: EPL 443 Holography and Optical Information Processing

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1. (a) Prove the convolution theorem:

$$\mathcal{F}\{\int_{-\infty}^{\infty} dx'\, g(x')\, h(x-x')\} = G(f_x)\, H(f_x).$$

In the above equation, G and H denote the Fourier transforms of g and h respectively. **(5 points)**

(b) Plot the function: $p(x) = \mathcal{F}^{-1}\{\mathrm{sinc}^2(f_x)\}$. Here $\mathrm{sinc}(u) = \frac{\sin(\pi u)}{\pi u}$. **(5 points)**

2. The Wigner distribution function is defined as: conjugate

$$W(x, f_x) = \int_{-\infty}^{\infty} d\xi\; g(\xi + x/2)\, g^*(\xi - x/2)\, \exp(-i2\pi f_x \xi).$$

(a) Evaluate $\int_{-\infty}^{\infty} dx\, W(x, f_x)$. **(5 points)** $g^*(\xi + x/2)$

(b) Evaluate $\int_{-\infty}^{\infty} df_x\, W(x, f_x)$. **(5 points)**

3. (a) State TRUE or FALSE with reasoning: For signals $g(x)$ bandlimited to frequencies $f_x : (-B, B)$

$$\int_{-\infty}^{\infty} dx'\, g(x')\, \mathrm{sinc}[2B(x - x')] = \frac{1}{2B}\, g(x).$$

(5 points)

(b) IIT Delhi central library needs a barcode scanner for reading barcodes on the books. A typical barcode is shown in the figure below. It has a width of 1 inch (2.54

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Figure 1: Typical barcode on a book. Barcode width = 1 inch.

cm) and the thinnest vertical bar is approximately 0.5 mm wide. The scanner device has a lens which images the barcode onto a linear array (single row) of detector pixels with suitable de-mgnification. Estimate the minimum number of pixels needed in the linear detector array for proper functioning of the barcode scanner. **(5 points)**

0.25

4. Consider the relation:

$$U(P_0) = \frac{1}{4\pi} \int\int_{S_1} ds \left(\frac{\partial U}{\partial n} G - U \frac{\partial G}{\partial n} \right).$$

Here S_1 is the surface $z = 0$ and U, G are scalar fields satifying the Helmholtz equation. Simplify the above relation with appropriate reasoning when

(a)

$$G = G_- = \frac{\exp(ikr_{01})}{r_{01}} - \frac{\exp(ik\tilde{r}_{01})}{\tilde{r}_{01}}$$

(5 points)

(b)

$$G = G_+ = \frac{\exp(ikr_{01})}{r_{01}} + \frac{\exp(ik\tilde{r}_{01})}{\tilde{r}_{01}}$$

(5 points)

In the above equations, r_{01} and $\tilde{r}_{01}$ denote distances between a point in the aperture (in the $z = 0$ plane) to two image points P_1 and $\tilde{P}_1$ respectively. (You may leave your answer in terms of the directional derivative.)