ELL311: Major Examination
Department of Electrical Engineering, IIT Delhi

Time: 2 hours **Maximum marks:** 40

- **Write your name and entry number on the uploaded answer script, failure to do which will fetch zero marks in the exam.**
- **Brevity in the answers will be given more credit.**
- **Make assumptions if required but state them clearly.**
- **Read the questions carefully before answering them. Answer all the parts of a question in one place. Untidy work will fetch a penalty of -2 marks.**

Undertaking: By attempting this paper you acknowledge that you will abide by the institute Honor Code and the code of conduct for this examination and can be held accountable as per rules established in case of any violation.

For all purposes ω denotes the angular frequency (rad/s) and f denotes the frequency (Hz), unless otherwise stated.

1. Answer the following questions briefly:
   (a) Mention one advantage and one disadvantage of FM over AM. [2]
   (b) When do we require the VSB modulation scheme as compared to a standard SSB modulation scheme for transmission? [1]
   (c) Draw the phasor plots for (i) NBFM (modulation index $\beta \ll 1$) (ii) AM signal (modulation index $0 < \mu < 1$), for a monotone message signal $m(t) = \cos(\omega_m t)$, assuming the carrier frequency to be ω_c. [2]

2. (a) Consider a message signal $m(t) = 10\cos(16\pi t)$. The signal is used to frequency modulate a carrier with carrier frequency, $f_c = 2$kHz. The resultant FM signal is passed through an ideal band-pass filter of total bandwidth 124 Hz centered at f_c. What are the frequencies present in the output signal of the band-pass filter, assuming $k_f \gg 1$? [2]
   (b) Consider an AM signal $x(t) = A_c(1 + \mu m(t))\cos(\omega_c t)$, $m(t) = \sum_{k=0}^{\infty} 2^{-k} \cos\left(\frac{\omega_m}{2^k} t\right)$, $\mu = 0.5$ and $A_c = 5$. Calculate the ratio of the message signal power to the total signal power. [3]

3. Justify whether the given statements are True or False with proper reasoning. No points will be awarded if reason is incorrect.
   (a) All second order ergodic processes are WSS. [2]
   (b) For a zero mean real WSS stochastic process $X(t)$, $S_X(\omega) \geq 0 \ \forall \ \omega$, where $S_X(\omega)$ is the power spectral density of $X(t)$. [3]
   (c) If P_X denotes the mean power of WSS process $X(t)$, with auto-correlation function $R_X(\tau)$, shown for $\tau \geq 0$ in figure 1, then $2 \leq P_X \leq 20$. [2]

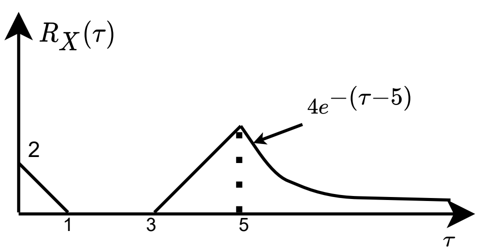

Figure 1: Figure for question 3(c)

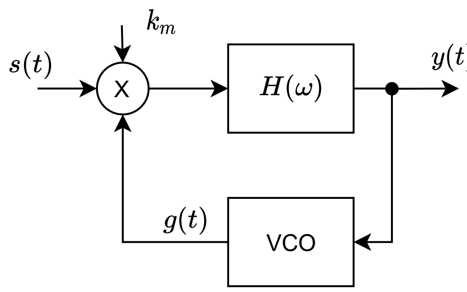

Figure 2: Figure for question 4

4. Consider an FM signal given by $s(t) = A_c \sin\left(\omega_c t + k_f \int\limits_0^t m(\lambda)d\lambda\right)$ demodulated using a PLL as shown in figure 2. The VCO output is given as $g(t) = A_v \cos\left(\omega_c t + k_v \int\limits_0^t y(\lambda)d\lambda\right)$ and $H(\omega)$ is a low pass filter. (i) Show that for the phase difference/error, $\phi_e(t)$ between $s(t)$ and $g(t)$, $\Phi_e(\omega) \approx \frac{\Phi_m(\omega)}{1 + L(\omega)}$, where $L(\omega) = \frac{K_0 H(\omega)}{j\omega}$, $\phi_m(t) = k_f \int\limits_0^t m(\lambda)d\lambda$, $K_0 = \frac{k_m k_v A_c A_v}{2}$. (ii) Expressing $Y(\omega)$ in terms of $\Phi_m(\omega)$ and $L(\omega)$, what happens to the output $y(t)$ when K_0 is sufficiently large? [6]

5. Consider a real discrete WSS stochastic sequence $X[n]$. Suppose we want to get the best estimate of $X[n]$ using a linear model based on the previous two samples, i.e. $X[n] = a_1 X[n-1] + a_2 X[n-2] + E[n]$, where $E[n]$ denotes the error. Find the values of a_1, a_2, given $R_X[k] = \alpha^{-|k|}$, $\alpha > 1$, where $R_X[k] = E\left[X[n]X[n-k]\right]$ is the auto-correlation function of $X[n]$. [5]

6. We want to transmit a real zero mean message signal $m(t)$, band-limited to $[-B, B]$ Hz, $B \ll f_c$ over a noisy channel using DSB-SC modulation scheme with carrier frequency f_c. The channel noise power spectral density given by $S_n(f)$ is shown in figure 3. Assuming the noise to be additive, derive the expression for the figure of merit, stating your assumptions clearly. [7]

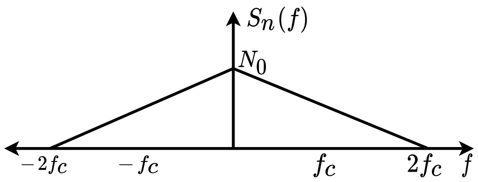

Figure 3: Figure for question 6

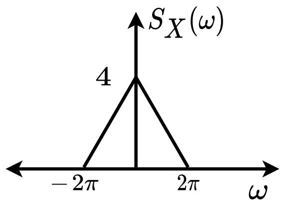

Figure 4: Figure for question 7

7. Let $X(t)$ denote a random process. If

$$Y = \int\limits_{-\infty}^{\infty} g(t)X(t)\,dt$$

such that Y is Gaussian distributed with $E[Y^2] < \infty$ for some function $g(t)$ then $X(t)$ is called a Gaussian random process. In particular, if we consider $g(t) = \delta(t - t_0)$, then $Y = X(t)\big|_{t=t_0}$. Now, given $X(t)$ is a WSS Gaussian random process, answer the following,

(a) If $E[X(t)] = 1$ and $S_X(\omega)$ is as shown in figure 4, then find the density function of the random variable $X_k = X(t)\Big|_{t=t_k}$. [2]

(b) Let $X(t)$ be a WSS Gaussian random process with $S_X(f)$ given as [3]

$$S_X(f) = \begin{cases} \frac{N_0}{2}, & \forall\ |f| \le 4B,\ B > 0 \\ 0, & \text{otherwise} \end{cases}$$

If $X(t)$ is passed through an LTI system with impulse response $h(t) = \frac{\sin(4\pi B t)}{\pi t}$ to obtain a process $V(t)$. Then for what values of $(t_2 - t_1)$ are $V_2 = V(t)\Big|_{t=t_2}$ and $V_1 = V(t)\Big|_{t=t_1}$ are **statistically independent**?