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HUL 320: Special Topics in Economics
Major Examination
1.5 hours

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Question 4 carries bonus points. Question 5 is to be attempted by those who missed minor 1 or 2.

1. Consider five lifeguard towers located along a beach and numbered sequentially from 1 up to 5. Two vendors, players 1 and 2, each selling soft-drinks can choose to be located next to some tower. There are 25 people located next to each tower and everyone would purchase soft-drink from the nearest vendor. In case of a tie, the demand is uniformly distributed, as usual. Each purchase yields a profit of 1 dollar. *(3+3+4)*

   (a) Write down the normal form of the game.

   (b) Are there any strictly dominated strategies?

   (c) Find the set of rationalizable strategies by IESDS.

2. Imagine a continuum of potential buyers, located on the line segment $[0, 1]$, with uniform distribution. Hence, the mass of buyers is $b - a$. Two firms, 1 and 2 are located at each end of the interval (firm 1 is at 0 and firm 2 is at 1). Each firm can choose its price p_i and each consumer goes to the seller who offers the "highest net value": a buyer who buys the product from firm i has a net value: $v - p_i - d_i$, where v is the value, p_i is the price and d_i is the distance between the buyer and firm i. Assume that the cost of production is 0. (2.5×4)

   (a) Assume that v is very large so that all the customers will be served by at least one firm and that some consumer $x^* \in [0, 1]$ is indifferent between firm 1 and 2. Find the best response for each firm.

   (b) If $v = 1$, find out the Nash equilibrium(a).

   (c) If the transportation cost becomes $\frac{d_i}{2}$ (i.e. net value for a buyer when he buys from firm i: $v - p_i - \frac{d_i}{2}$), find out the Nash equilibrium(a).

   (d) Generally if the transportation cost is $\alpha.d_i$, $\alpha \in [0, \frac{1}{2}]$, what happens to Nash equilibrium(a) as $\alpha \to 0$? What is the intuition?

3. In an all pay auction, each bidder loses its bid irrespective of the outcome. Consider an all pay auction for a good with value 1 to each of the two bidders. Each bidder i can choose to bid from the interval $s_i = [0, 1]$. Players care about the expected value they end up with at the end of the game.

(3+3+4)

(a) Model this auction as a normal-form game.

(b) Does this game have any pure strategy Nash equilibria?

(c) Show that this game cannot have a Nash equilibrium in which each player is randomizing over a finite number of bids.

4. State if the following statements (any two) are True or False with a very short argument:

(5)

(a) In common value auctions, values of the players are common knowledge.

(b) In finitely repeated prisoner's dilemma game, cooperation can be attained as Nash equilibrium.

(c) In correlated equilibrium outcomes, the players choose their strategies in consultation with some other players.

5. You and a friend are in an Italian restaurant, and the owner offers both of you an 8-slice pizza for free under the following condition. Each of you must simultaneously announce how many slices you would like; that is, each player $i \in \{1, 2\}$ names his desired amount of pizza, $0 \leq s_i \leq 8$. If $s_1 + s_2 \leq 8$ then the players get their demands (and the owner eats any leftover slices). If $s_1 + s_2 > 8$, then the players get nothing. Assume that you each care only about how much pizza you individually consume, and the more the better.

(5+5)

(a) Write each player's best-response correspondence.

(b) Is (Are) there any pure-strategy Nash equilibrium(a)?