IIT Delhi: HUL320B-odd semester 2022-23
Selected Topics in Economics: Major exam

Time Allowed: 2 hours; Marks: 50 (marks breakup is given at the end of each question).

Instructions: There are 4 questions. But Marks will be determined by taking the best 3 out of 4 answers.

(1) A market for samosas has inverse demand $P = 1 - Q$. There are n samosa-selling firms operating within the boundaries of a town, each with unit cost of production c, where $0 < c < 1$. The firms decide their quantities as in a Cournot Oligopoly.
(a) Compute the quantity that each firm will produce in a Cournot-Nash equilibrium. Compute the industry output Q, price P, and each firm's equilibrium profit.
(b) The Municipal Authority of the town decides to tax the sales of each firm. It imposes a tax amount t per unit sold, where $0 < t < 1 - c$.
(b.1) What quantity will each firm now produce in a Cournot-Nash equilibrium. And what is the total, industry output?
(b.2) Work out the per-unit tax t that will maximize the tax revenue of the Municipal Authority. (Tax Revenue $= tQ(t)$, where $Q(t)$ is the industry output in Cournot-Nash equilibrium, when there is a tax of t per unit output).
What is the Municipal Authority's maximal Tax Revenue from imposing the above level of t? What is the tax paid by each firm?
(c) Suppose that instead of taxing the firms, the Municipal Authority asked each firm to pay it a lumpsum amount T, in exchange for a license to permit the firm to operate. Determine some level at which T can be fixed such that firms find it profitable to operate, and the Municipal Authority's Tax Revenue (which now will equal nT) is higher than in (b.2) above. Note that you must work out / argue out what the Cournot-Nash equilibrium quantities and profits are in this question.

Marks (a) 8; (b) 6; (c) $2\frac{2}{3}$

(2). Consider the following Hotelling model. Consumers are uniformly distributed on an interval $[0, 1]$, Firm 1 is located at point 0, Firm 2 is located at point 1. The Firms sell a homogeneous product; Firm 1's unit cost is c, $0 < c < 1$, Firm 2's unit cost is 0. Each consumer wants 1 unit of the good and values the good at v, which is high enough that every consumer will buy from one of the 2 firms. If a consumer located at point x buys from Firm 1

at price p_1, then he or she pays an effective price (or hedonic price) $p_1 + x$ for Firm 1's product, where x is the transport cost; and if he or she buys from Firm 2 at price p_2, the effective price is $p_2 + (1 - x)$, where $(1 - x)$ is the transport cost. (So, in this setting, distance of the consumer from the firm equals their transport cost).
(a) Consider first a game in which the 2 Firms simultaneously set prices (p_1, p_2), and then each consumer buys the product that is cheaper in terms of effective price (price plus transport cost; an indifferent consumer can buy either product). Compute the Nash Equilibrium prices, market shares, and profits of the 2 Firms.
(b) Now consider an alternative situation, in which a firm can charge each consumer a different price (i.e. perfect price discrimination). Compute equilibrium profits and market shares. (Hint: What would the prices offered to the indifferent consumer be?)

Marks: (a) 10; (b) $6\frac{2}{3}$

(3) Suppose two friends visiting Italy go out to eat. Each individual i has utility for slices of pizza, x_i, and money, m_i, given by

$$u_i(x_i, m_i) = 5x_i - 0.5x_i^2 + m_i, i = 1, 2$$

where m_i is in Euros. Suppose that each individual has 10 Euros in her wallet and that the price of pizza is 2 Euros per slice.
(a) If each person pays for her own pizza slices, how many slices will she eat? How much net consumer surplus will each person get? (Net consumer surplus is the change in utility from the purchase and consumption of pizza slices relative to not purchasing or consuming any.)
(b) If the friends decide in advance, instead, that they will each pay half of the total bill, how many slices will each eat, if their choices of slices are now determined in a Nash Equilibrium? How much consumer surplus will each individual get?

Marks: (a) $6\frac{2}{3}$; (b): 10

(4) (a) Two firms, each with unit cost c and unlimited capacity, sell a good whose inverse demand curve is given by $P = A - Q$, where Q is quantity and $A > c + 1$. The firms compete as Bertrand competitors by simultaneously setting prices; the lower-priced firm gets the entire demand, whereas if both firms charge the same price, they get half the demand each.

The firms can *only set prices as positive integers* as 1 Rupee is the unit of currency. Find a Nash equilibrium in prices. You must show your working.

(4) (b) Two firms operate in a market with inverse demand curve $P = 1 - Q$. Both firms have unit cost c, $0 < c < 1$. Time periods are $t = 0, 1, 2, \ldots$; the firms set quantities q_{1t}, q_{2t} in each period t, with price then being equal to $P_t = 1 - q_{1t} - q_{2t}$. In this infinitely-repeated game, the firms discount the future using discount factor δ, $0 < \delta < 1$.

Let q_m be the profit-maximizing quantity that a monopoly would produce, and let q_1^c, q_2^c be the Cournot-Nash Equilibrium outputs. Consider the following strategy profile: Start out by each firm producing $(\frac{1}{2})q_m$ each, and do so each period, unless any firm deviates by producing some other quantity. Following any such deviation, both firms produce the Cournot-Nash outputs forever.

What is the smallest discount factor δ for which the above strategy profile is a subgame-perfect equilibrium?

Marks: (a) 10; (b) $6\frac{2}{3}$