IIT Delhi, Department of Mathematics

MAL335: Differential Equations

Minor-1 Max. Marks 20 Time: 1 hour

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1. Determine if the following functions are linearly dependent or independent on $(-\infty, \infty)$:
   (a) $\sin 2x, e^{i2x}, 5\cos 2x$ (b) $x^2, e^{2x}, x|x|$. [4]

2. Find all solutions of the following equations:
   (a) $y''' - y' = x$ (b) $y^{(4)} - 4y''' + 6y'' - 4y' + y = e^x$. [4]

3. (a) Let ϕ_1, ϕ_2, ϕ_3 be any three solutions of $L(y) = y''' + a_1 y'' + a_2 y' + a_3 y = 0$ on an interval I. Prove that the Wronskian $W(\phi_1, \phi_2, \phi_3)(x) \neq 0 \quad \forall \quad x \in I$ if and only if ϕ_1, ϕ_2, ϕ_3 are linearly independent on I. (b) Are these two functions $x^2, x|x|$ linealy independent on $(-\infty, \infty)$? (c) Compute the Wronskian of these two functions. (d) Do the results of (b) and (c) contradict the result in (a) for two functions? Justify your answers. [6]

4. Suppose the constants $a_1, a_2, \cdots, a_n$ in the equation $L(y) = y^{(n)} + a_1 y^{(n-1)} + \cdots + a_n y = 0$ are all real. Let $\phi_1, \phi_2, \cdots, \phi_n$ be n linearly independent real-valued solutions of the above $L(y) = 0$. (a) Prove that every real-valued solution is a linear combination of these n solutions with real-coefficients. (b) Prove that every solution that satisfies real initial conditions is real-valued solution. [6]