Department of Mathematics
II Semester 2013-2014
MAL 342 Analysis and Design of Algorithms
Minor II   Weightage 25%
Date 23.3.14   Time 8 -9 A.M

Q1. Define the maximum weight independent set problem in a weighted Matroid. Show that the minimum spanning tree problem on G can be modeled as a maximum weight independent set problem in an appropriate Matroid. Justify your answer. [2+6]

Q2. A product of matrices is fully paranthesized if it is either a single matrix or the product of two fully parenthesized matrix products surrounded by parenthesis. (For example (A(B(CD))) is fully parenthesized. There are four different ways of parenthesized the product of four matrices). Given a chain $A_1,A_2,....A_n$ of n matrices, where for i=1,2,...,n, matrix A_i has dimension $p_{i-1} \times p_i$, the **Chained Matrix Multiplication problem** is to fully parenthesize the product $A_1A_2...A_n$ in a way that minimizes the number of scalar multiplications. (for example, given the product $A_1A_2A_3$, with $p_0=3, p_1=4, p_2=5$ and $p_3=6$, $(A_1(A_2A_3))$ takes 120+72=192 scalar multiplication).

Consider the following two Greedy algorithms for chained matrix multiplication problem.

10x2   2x3   3x5   4x5

1. At each step compute the cheapest multiplication.   8
2. At each step compute the most expensive multiplication.

For each of the above algorithms, either prove that the algorithm minimizes the total number of multiplication or show that the algorithm does not minimize the total number of multiplication by producing a counter example. [2+2]

Q3. Describe the Kruskal's MST Algorithm. Prove that Kruskal's MST algorithm always produces an MST of a weighted connected graph. [2+6]

Q4. Let G=(V,E) be a weighted connected graph with weight function $w: E \to R^+$. Let e be a minimum cost edge of G. Prove that there is a minimum cost spanning tree T of G containing e. [5]
