

**Department Of Mathematics  
Indian Institute Of Technology Delhi  
MAJOR – TEST**

Time : 2 Hours

MAL 382 – Theory of Automata

Full Marks - 40

Date: May 5, 2014

Q1. Give the scheme, describing the functions needed and how they are applied, to design a TM that takes as input an integer  $n$ , in unary, and returns 0 or 1 depending upon whether the number is a prime or composite.

[9]

Q2. Consider the language  $\{0^n 1^n \mid n > 0\}$ .

a) Design a standard TM to accept the above language. What is its complexity?

b) Design a Semi-infinite tape Automata for accepting the above language - starting with the configuration given below:

- the tape has two tracks, and it is infinite only on the right side.
- all 0's are in the upper track and the 1's in the lower track.
- initially the tape head is at the extreme left end of the tape.

[3 + 6 = 9]

Q3. a) Write a grammar for generating the language  $\{a^n b^{2n} c^{3n} \mid n > 0\}$

b) Distinguish between Universal TM and Universal Language.

c) If  $L_1$  is recursive and  $L_2$  is recursively enumerable, then what are the languages  $L_1 L_2$  and  $L_2 L_1$ ? Justify your answer.

[4 + 3 + 3 = 10]

Q4. a) What is the "Halting Problem" with respect to Turing Machines? Is it decidable? Justify your answer.

b) Distinguish between PC and MPC problems. Show that an instance of an MPCP is reducible to an instance of a PCP.

[6 + 6 = 12]