Department Of Mathematics
Indian Institute Of Technology Delhi
MAL 390 – Statistical Methods and Algorithms
Major Test – 01-05-2015

Time: Two Hour

Total Marks: 30

Q1. Consider the following bi-variate data table:

	16	1	1	9	16	9	4	4	16	9	4	1	4	16	9	1
X	4	1	1	3	4	3	2	2	4	3	2	1	2	4	3	1
Y	1	1	4	1	4	2	4	3	3	4	2	3	1	2	3	2
	4	1	4	3	16	6	8	6	12	12	4	3	2	8	9	2

(a) Fit the two regression lines $Y = a + bX$ and $X = c + dY$ using the Least Square method and solving Normal equations.

(b) What is the angle between the two lines? Justify your answer.

(c) Use Least square theory to find the regression coefficients Y is regressed on $\alpha + \beta X$ for two non-zero constants α and β.

[3 + 1 + 2 = 6]

Q2. (a) Explain the Wilcoxon Signed Rank test for comparing the central tendencies of two populations.

(b) Explain how Kendall's Tau is used for computing the correlation between bi-variate observations.

(c) Compute the value of Tau for the following data:

X	4	1	5	3	2
Y	1	3	4	2	5

[2 + 2 + 2 = 6]

Q3. (a) Prove or Disprove: If T is the Maximum Likelihood estimator for θ, and Ψ is a strictly monotonic function, then $\Psi(T)$ is Maximum Likelihood estimator for $\Psi(\theta)$.

(b) Prove or Disprove: If T is sufficient for θ, then T is also Maximum Likelihood estimator for θ.

(c) Find the Maximum Likelihood estimate for $\frac{1}{\theta}$ from one observation on the random variable $X \sim \theta\,(1 - \theta)^{x-1}$, $x = 1, 2, 3, \ldots\ldots$

[2 + 2 + 2 = 6]

Q4. (a) Distinguish between Most Powerful and Uniformly Most Powerful Critical regions while testing a statistical hypothesis.

(b) Use Neyman-Pearson Lemma to construct the Most Powerful Critical Region for testing $H_0 : \theta = \theta_0$ vs. H1: $\theta = \theta_1$ where $\theta_0 < \theta_1$, based on a sample of size n taken from $N(0, \theta)$, at 95% level of confidence. Note: θ stands for the variance.

[2 + 4 = 6]

$z(n) = az(n-1)$

Q5. (a) Describe the Lehmer's algorithm for generating random numbers.

(b) Suppose the following 10 random numbers are generated in the range 1 - 100:

15, 92, 81, 23, 18, 73, 64, 95, 41 and 32.

Use the above numbers to generate 10 random numbers from N(0,1) population. Justify your answer.

[3 + 3 = 6]

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