MAL-518 Methods of Applied Mathematics

Department of Mathematics, IIT Delhi

Major, (May 2015)

Time: 2 Hours

Max. Marks: 50

1. (a) Solve the following problem by using method of separation of variables:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\left(\frac{\partial^2 u}{\partial t^2} + 2k\frac{\partial u}{\partial t}\right), \quad 0 < x < a, t > 0, k > 0,$$

$$u(0,t) = u(a,t) = 0, t > 0, u(x,0) = f(x), \frac{\partial u}{\partial t}(x,0) = 0, 0 < x < a,$$

(b) Find product solutions of the following problem

$$\frac{\partial^4 u}{\partial x^4} = -\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}, \quad 0 < x < a, t > 0,$$

$$u(0,t) = u(a,t) = 0, \frac{\partial^2 u}{\partial x^2}(0,t) = 0, \frac{\partial^2 u}{\partial x^2}(a,t) = 0.$$

[4+4]

OR

1. State and prove the theorem of successive approximation. [8]

2. (a) Solve the potential equation in the slot $0 < x < a, y > 0$ under the boundary conditions:

$$\frac{\partial u}{\partial x}(0,y) = 0, u(a,y) = 0, u(x,0) = 1.$$

(b) Prove that $\frac{d}{dx}(x^{\mu}J_{\mu}(x)) = (x^{\mu}J_{\mu-1}(x))$, where $J_{\mu}(x)$ is Bessels function of order μ. [6+2]

3. (a) State and prove the theorem of successive substitution.

(b) Obtain the resolvent kernel $R(s,t;\lambda)$ of the given kernel $K(s,t) = \cos(s+t)$ $[0, \pi]$ [8+4]

4. (a) Write the statement of Fredholm first and third theorems. Give the proof of Fredholm first theorem only.

(b) Determine the unique solution of following integral equation

$$\phi(x) = e^{2x} + \int_0^1 (xe^t + te^x)\phi(t)dt.$$

(c) Is it true or false that the following integral equation

(i) has no solution for $f(x) = x$,

(ii) posseses infinitely many solutions when $f(x) = 1$,

$$\phi(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)\phi(t)dt.$$

Justify your answers. [illegible]