

$X_{(n)} \leq L$
 $0 < x < 1$

Department of Mathematics
MAL 522 (Statistical Inference)
Major Test (II Semester 2014 - 2015)

Maximum Marks: 50

Answer All Questions

Time: 2 hours

1. A random sample of size n is obtained from a uniform distribution on the interval $(0, 1)$. Show that $\frac{X_{(1)}}{X_{(n)}}$ and $X_{(n)}$ are independent random variables. (7 marks)

2. Let (X_1, X_2, \dots, X_n) be a random sample from pdf $f(x; \theta) = \theta x^{\theta-1}$, $0 < x < 1$, $\theta > 0$. Is $\sum_{i=1}^n X_i$ sufficient for θ ? Find a complete sufficient statistic for θ . (3 + 3 marks)

3. The average weight of 25 new born babies is 2.78 kgms with a standard deviation of 0.72 kgms. Construct a 99% confidence interval for the true mean weight of such babies under the assumption that the population follows normal distribution. (6 marks)

4. It is desired to test the hypothesis $\mu = 0$ against the alternative $\mu > 0$ on the basis of a random sample of size 9 from a normal population with variance $\sigma^2 = 1$. Show that $\bar{X} > 0.78$ is the critical region of size $\alpha = 0.01$. (6 marks)

5. State Neyman-Pearson lemma. Find Neyman-Pearson size α test of $H_0 : \beta = 1$ against $H_1 : \beta = \beta_1 (> 1)$ based on a sample of size 1 from $f(x, \beta) = \beta x^{\beta-1}$, $0 < x < 1$. (3 + 4 marks)

6. Show that $f(x, \theta) = \frac{\theta}{x^2}$, $x > \theta$, $\theta > 0$ has MLR in $X_{(1)}$ and obtain a UMP test for $H_0 : \theta \geq \theta_0$ against $H_1 : \theta < \theta_0$. (3 + 3 marks)

7. Suppose we believe that the life time T of light bulbs is exponential distribution with parameter 0.005. We obtain a sample of 150 bulbs, test them, and record their burning time. The data are given in following table. Test the hypothesis that the data represent a sample from an exponential distribution with parameter 0.005 at 1 percent level of significance. ($\chi^2_{3,0.01} = 11.345$)

Events	Number of occurrence
$0 < T < 100$	47
$100 \leq T < 200$	40
$200 \leq T < 300$	35
$(300 \geq T)$ $T \geq 300$	28

(6 marks)

8. A vote is to be taken among the students of MAL 250 course in two sections to determine whether a quiz should be conducted. To determine if there is a significant difference in the proportion of two section voters favouring the proposal, a poll is taken. If 24 out of 40 students in section I favour the proposal and 24 out of 50 students in section II favour it, would you agree that the proportion of section I student voters favouring the proposal is higher than the proportion of section II student voters? Use an $\alpha = 0.05$ level of significance. (6 marks)

(Use $Z_{0.01} = 2.325$; $Z_{0.05} = 1.645$; $t_{24,0.005} = 2.757$)