Nonlinear Optimization: Major Exam for MCL261
November 20, 2017
Total marks: 14

1. (2 marks) Consider the following minimization problem:

$$\underset{x}{\text{minimize}}\ a^T x$$
$$\text{subject to: } c_i^T x \geq b_i, \quad i = 1 - m$$

Prove that this is a convex optimization problem.

2. (3 marks) Consider the following problem:

$$\underset{x}{\text{minimize}}\ (x-p)^T(x-p)$$
$$\text{subject to: } c^T x = b$$

Here x, p, and $c \in \mathbb{R}^n$.

(a) Find a stationary point for this problem and express it in terms of p, c, and b.

(b) If we write c as $[c_1, c_2, ..., c_n]^T$, find the null space matrix Z for the constraint matrix c^T in terms of the elements of c.

3. (3 marks) Consider the following problem:

$$\underset{x \in \mathbb{R}^3}{\text{minimize}}\ f(\mathrm{x})$$

Let the Hessian of $f(\mathrm{x})$ be given by $\nabla^2 f(\mathrm{x}) = \begin{pmatrix} 6 & 4 & 3 \\ 4 & 1 & 5 \\ 3 & 5 & 3 \end{pmatrix}$, where $\nabla^2 f(\mathrm{x})$ is used to compute a descent direction at x. Find the matrix E to be added to $\nabla^2 f(\mathrm{x})$ to guarantee descent at x. In your LDL^T factorization process, replace each negative diagonal entry by rounding up its absolute value to the nearest integer.

4. (2 marks) Show that the following problem can be solved using just one iteration of Newton's method for minimization, and find the solution. Assume that A (an n x n matrix) is positive definite and $x, b \in \mathbb{R}^n$.

$$\underset{x}{\text{minimize}}\ \frac{1}{2}x^T A x - b^T x$$

$\begin{bmatrix}1 & 1 & \\ & 2 & \\ & & 3\end{bmatrix}\begin{bmatrix}1 & & \\ 1 & 2 & \\ 0 & & 3\end{bmatrix}.$

$x_{ij} - P_{ij}$

MCL261 Major

Part (A)

Max marks: 26 | Date: 20th November 2017

Instructions:

- The method used is very important and clearly state all the assumptions made
- Please return the question paper with the answer sheet
- **Part (A) and (B) should be attempted on separate answer sheets. Please clearly specify Part (A) / (B) on the answer sheet**
- **Part (A) contains 4 questions**

1. Solve the following LP: (7)

$$Maximize\ Z = 5x_1 + 2x_2 - 2x_3$$ → λ

s.t. $x_1 + x_2 + x_3 \geq 8,\ 2x_1 + x_2 \leq 14, x_1 + x_3 \leq 16, x_1, x_2, x_3 \geq 0$ → λ

μ

2. Solve the following transportation problem: (6)

	Destination 1	Destination 2	Destination 3	Destination 4	Supply
Factory A	10	15	9	12	8
Factory B	5	10	8	15	7
Factory C	15	10	12	12	10
Demand	5	9	6	5	

The cost of transportation from Factory to Destinations is given in each cell above, and the demands and supplies for each Destination and Factory are also given. Find the optimal solution for the problem.

3. A group of 4 friends would like to go to watch a movie. They can either go by a bus or an auto. However, in an auto only 2 of them can travel together, while in a bus, any number of them can travel together. They would travel in whichever mode of transport arrives first. The arrival rates of both auto and bus are distributed exponentially with parameters λ_{auto} and λ_{bus} respectively. The bus always takes 30 min to reach the movie hall, while the auto always takes 20 min. What is the probability that all of them would reach at the EXACTLY the same time? (6)

4. A café with a single server sells tea and coffee, with separate queues for tea and coffee. There can be at most one person in each queue at any given instance. The arrival rates of customers demanding tea and coffee are both, distributed exponentially with arrival rates λ_t and λ_c respectively. The server can prepare only one drink at a time, and the preparation rates are also exponentially distributed with mean preparation rate μ for both tea and coffee. It is known that $\mu = (\lambda_t + \lambda_c)/2$. In case both the queues have a single person each, the owner needs to decide whether to serve tea or coffee, in order to minimize the number of customers lost. Let us denote these by

Scenario T: Server serves tea in case both queues have a single person

Scenario C: Server serves coffee in case both queues have a single person

(a) Find the long run probability of both queues being empty in case of Scenario T (2)

(b) Find the rate of customers lost in case of Scenario T (in terms of $\lambda_t, \lambda_c, \mu$) (3)

(c) Find the rate of customers lost in case of Scenario C (in terms of $\lambda_t, \lambda_c, \mu$) (1)

(d) Using parts (b) and (c), find a condition in terms of λ_t, λ_c, such that Scenario T would minimize the number of customers lost (1)

State (1, 0,