MCL261 Minor 1

Part (A)

Max marks: 13

Date: 31st August 2017

Instructions:

- The method used is very important and clearly state all the assumptions made
- Please return the question paper with the answer sheet
- **Part (A) and (B) should be attempted on separate answer sheets. Please clearly specify Part (A) / (B) on the answer sheet**

$x_1 = 1,\ x_2 = x_3 = 0.$

1 (a) Find the optimal values of x_1, x_2, x_3: $\min. (x_1 - 1)^2 + x_2^2 + x_3^2$ s.t. $x_1 + x_2 + x_3 \le 1,\ x_i \ge 0$ (3)

(b) When will the following problem always have a solution?

$$\min cX,\ s.t.\ AX \le b,\ X \ge 0$$

(c) Solve the following problem:

$$\text{Minimize } Z = x_1 + 2x_2 - x_3 \quad s.t.$$

$$x_1 + 3x_2 + x_3 = 20$$

$$x_1 - x_2 + x_3 = 0$$

$$x_1 + x_2 + x_3 = 10$$

$$2x_1 + 6x_2 + 2x_3 = 40$$

$$x_1, x_2, x_3 \ge 0$$

2. Solve the following LP using the Big M method: (6)

$$Maximize\ Z = 3x_1 + 5x_2$$

$Z = 3x_1 + 5x_2 + MR_1$

s.t. $x_1 \le 4$

$2x_2 \le 12$

$3x_1 + 2x_2 = 18$

$x_1, x_2 \ge 0$

$x_1 + S_1 = 4$

$x_2 + S_2 = 12$

$3x_1 + 2x_2 + R_1 = 18$

3. Consider the following problem: (4)

$$Maximize\ Z = 5x_1 + 3x_2 + 4x_3, \text{ subject to}$$

$$2x_1 + x_2 + x_3 \le 20$$

$$x_1 + x_2 + 2x_3 \le 30$$

$$x_1, x_2, x_3 \ge 0$$

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You are given the information that the **non-zero variables in the optimal solution are x_2 and x_3. Use this information to** solve the problem optimally.

Basic variables.

min

Nonlinear Optimization: Minor 1 Exam for MCL261
August 30, 2017
Total marks: 7

1. Consider the least squares data-fitting problem for a linear regression model:

$$\underset{\mathbf{x}}{\text{minimize}} \;\; f(\mathbf{x}) = \sum_{i=1}^{N} [y_i - (x_0 + x_1 t_i)]^2$$

Here, $\mathbf{x} = [x_0 \; x_1]^T \in \mathbb{R}^2$ represents the vector of model parameters, N is the size of the sample (number of data points), the y_i represent the observed data for the dependent variable, and the t_i represent the observed data for the independent variable.

Is this a convex optimization problem? Show your work to prove yes/no. (4 marks)

2. Prove that the intersection $\cap_{i=1}^{N} S_i$ of finitely many convex sets $S_i, i = 1, 2, ..., N$ is convex. (1.5 marks)

3. Can you use Newton's method for root-finding to calculate the reciprocal of a scalar $x \in \mathbb{R}$? If so, write down (a) the function that you would find the roots of, and (b) the expression for the k^{th} iteration that yields the value of the $k + 1^{th}$ estimate of the root. (1.5 marks)

$\nabla^2 f(x) \sqsupset 0.$

$X. = X_k - \frac{f(k)}{f'(k)}.$

$2 \rightarrow \frac{1}{x}$

$X_0 = \frac{1}{X}$

$f(x) = \frac{1}{X}$

5

$f(x_0) =$

$X_0 -$

$X_0 = 1.$

$\log |x|.$

$\frac{1}{x^2}$

$x +$

$\frac{x f(x)}{} = \frac{f'(x)}{}$

$f'(x) = -\frac{1}{x^2}$

$\frac{1}{X}$

$\frac{1}{2} e^{x^2}$ $\Rightarrow$