

MCL261 Minor 1

Part (A)

Max marks: 13

Date: 31st August 2017

Instructions:

- The method used is very important and clearly state all the assumptions made
- Please return the question paper with the answer sheet
- Part (A) and (B) should be attempted on separate answer sheets. Please clearly specify Part (A) / (B) on the answer sheet

$$x_1 = 1, x_2 = x_3 = 0.$$

1 (a) Find the optimal values of x_1, x_2, x_3 ; $\min. (x_1 - 1)^2 + x_2^2 + x_3^2$ s.t. $x_1 + x_2 + x_3 \leq 1, x_i \geq 0$ (3)

(b) When will the following problem always have a solution?

$$\min cX, \text{ s.t. } AX \leq b, X \geq 0$$

(c) Solve the following problem:

$$\text{Minimize } Z = x_1 + 2x_2 - x_3 \quad \text{s.t.}$$

$$x_1 + 3x_2 + x_3 = 20$$

$$x_1 - x_2 + x_3 = 0$$

$$x_1 + x_2 + x_3 = 10$$

$$2x_1 + 6x_2 + 2x_3 = 40$$

$$x_1, x_2, x_3 \geq 0$$

2. Solve the following LP using the Big M method:

$$\text{Maximize } Z = 3x_1 + 5x_2$$

$$\text{s.t. } x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 = 18$$

$$x_1, x_2 \geq 0$$

$$Z = 3x_1 + 5x_2 + MR_1 \quad (6)$$

$$x_1 + S_1 = 4$$

$$x_1 + S_2 = 12$$

$$3x_1 + 2x_2 + R_1 = 18$$

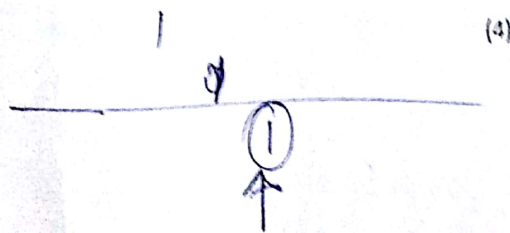
3. Consider the following problem:

$$\text{Maximize } Z = 5x_1 + 3x_2 + 4x_3, \text{ subject to}$$

$$2x_1 + x_2 + x_3 \leq 20$$

$$x_1 + x_2 + 2x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$



You are given the information that the non-zero variables in the optimal solution are x_2 and x_3 . Use this information to solve the problem optimally.

Basic variables.

min \uparrow

Nonlinear Optimization: Minor 1 Exam for MCL261
 August 30, 2017
 Total marks: 7

1. Consider the least squares data-fitting problem for a linear regression model:

$$\underset{x}{\text{minimize}} f(x) = \sum_{i=1}^N [y_i - (x_0 + x_1 t_i)]^2$$

Here, $x = [x_0 \ x_1]^T \in \mathbb{R}^2$ represents the vector of model parameters, N is the size of the sample (number of data points), the y_i represent the observed data for the dependent variable, and the t_i represent the observed data for the independent variable.

Is this a convex optimization problem? Show your work to prove yes/no. (4 marks)

2. Prove that the intersection $\cap_{i=1}^N S_i$ of finitely many convex sets $S_i, i = 1, 2, \dots, N$ is convex. (1.5 marks)

3. Can you use Newton's method for (root-finding) to calculate the reciprocal of a scalar $x \in \mathbb{R}$? If so, write down (a) the function that you would find the roots of, and (b) the expression for the k^{th} iteration that yields the value of the $k + 1^{th}$ estimate of the root. (1.5 marks)

$\nabla^2 f(x) = 0$

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

2 \rightarrow $\left(\frac{1}{x}\right)$

$$x_0 = \frac{1}{x}$$

$$f(x) = \frac{1}{x}$$

(5)

$$f(x_0) =$$

$$x_0 - \frac{f'(x_0)}{f''(x_0)}$$

✓

$$x_0 = 1$$

1

$$\log|x|$$

$$\frac{1}{x^2}$$

x_+

$$x \frac{f'(x)}{f(x)} = \frac{f''(x)}{f'(x)}$$

$$f'(x) = -\frac{1}{x^2}$$

$$\frac{1}{x}$$

$$\frac{1}{2} e^{x^2} \Rightarrow$$