MCL261 Minor 2

Part (A)

Max marks: 13 Date: 6th October 2017

Instructions:

- The method used is very important and clearly state all the assumptions made
- Please return the question paper with the answer sheet
- **Part (A) and (B) should be attempted on separate answer sheets. Please clearly specify Part (A) / (B) on the answer sheet**

1. Consider the following transportation problem:

	Destination 1	Destination 2	Destination 3	Destination 4	Destination 5	Supply
Factory A	7	5	4	3	2	20
Factory B	6	5	3	5	4	40
Factory C	2	7	4	6	3	80
Demand	10	20	10	40	60	

The cost of transportation from Factory to Destinations is given in each cell above, and the demands and supplies for each Destination and Factory are also given.

(a) Use the Vogel's Method to get an initial BFS for the above problem. (2)

(b) Obtain the optimal solution for the problem, using the solution from Vogel's method. (5)

2. Consider a single server queueing system, with at most 2 people in the <u>queue</u>. The service time is an exponential distribution with parameter μ. In addition, some people may also leave the queue without being serviced. This has been observed to follow an exponential distribution with parameter ξ. The arrival rate follows an exponential distribution with parameter = $\lambda/(n+1)$, where n denotes the number of people currently in the system.

Find the stationary distribution for the system using **first principles only**. (4)

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$x_0 - \frac{f(x_0)}{f'(x_0)}$

Nonlinear Optimization: Minor 2 Exam for MCL261
October 6, 2017
Total marks: 7

1. (2 marks) Consider the following function: $(x-1)(x-2)^4$ $\quad f(1.5) = (0.5)^5$

$$f(x) = (x-2)^4 + (x-2)^5 \quad f'(x) = (x-2)^4 + 4(x-1)(x-2)^3$$

(a) What are the roots of this function? $\quad f'(0.5) = 5(0.5)^4$

(b) Comment on the rates at which Newton's method is likely to converge to the different roots of this function.

(c) Starting at $x_0 = 1.5$, which root will Newton's method converge to?

2. (4 marks) Consider the following problem:

$$\underset{\mathbf{x}}{\text{minimize}} \;\; f(\mathbf{x}) = (x_1 - x_2)^2 + x_1^3 + x_2^2$$

Here $\mathbf{x} \in \mathbb{R}^2$. Answer the following questions:

(a) Find the stationary point(s) of $f(\mathbf{x})$.

(b) Is the second-order necessary condition for a local minimizer satisfied at the stationary point(s) identified? Show your work to prove yes/no.

(c) Are any of the stationary point(s) local optima? Show your work to prove yes/no.

(d) Are any of the stationary point(s) global optima? Show your work to prove yes/no.

3. (1 mark) (a) Using the first-order necessary condition for (local) optimality, derive the second-order necessary condition for (local) optimality for an unconstrained nonlinear maximization problem.

(b) Also show how the second-order sufficient condition for optimality guarantees that a stationary point is a local maximizer for an unconstrained nonlinear maximization problem.

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$b^2 > 4ac$

$b^2 - 8 \quad\quad b^2 - ac$

$2y_2 - 2y_1 + 4y_2 \quad 36 - 8$

$16 - (4)(1)(-4)$

$-2y_2y_1 - 2y_1y_2 + 4y_2^2 \geq 8$

$\boxed{4y_2(y_2 - y_1)}$ $\quad 9 - \quad 1 \pm \sqrt{32}$

$9 - (4)(2) \quad 2 \pm \sqrt{8}$

$\frac{3 \pm \sqrt{36 - (4)(4)}}{2} \quad 9 - 4$