MCL361 Minor Exam (18/03/2021)

For Questions 1-7, consider the EPL model from Assignment 1. Assume the production rate to be P units per unit time. Let the lot (order) size be denoted by y units. Assume the demand rate to be D units per unit time. Assume that $P > D$. Let K be the setup cost associated with the placement of an order given in INR per order. Let h be the holding cost in INR per inventory unit per unit time. The reorder point should be set to zero as in the classical EOQ model. Assume that as soon as the order is placed, production starts. Also assume that as soon as a unit is produced, it arrives at the inventory facility; there is no delivery lag.

1) (1 mark) Similar to EOQ model, plot the inventory level against time assuming there was no unit in the inventory facility to begin with. Indicate the slopes of all the lines shown in the plot.
2) (0.5 mark) What is the time required to produce a single lot?
3) (0.5 mark) What is the maximum inventory size that can occur in EPL model?
4) (1 mark) What is the time period of each inventory cycle?
5) (1.5 marks) What is the holding cost per cycle?
6) (1 mark) What is the total inventory cost per unit time TCU(y)?
7) (1 mark) What lot size minimizes TCU(y)? Do justify that the obtained stationary point is indeed a minimum point.

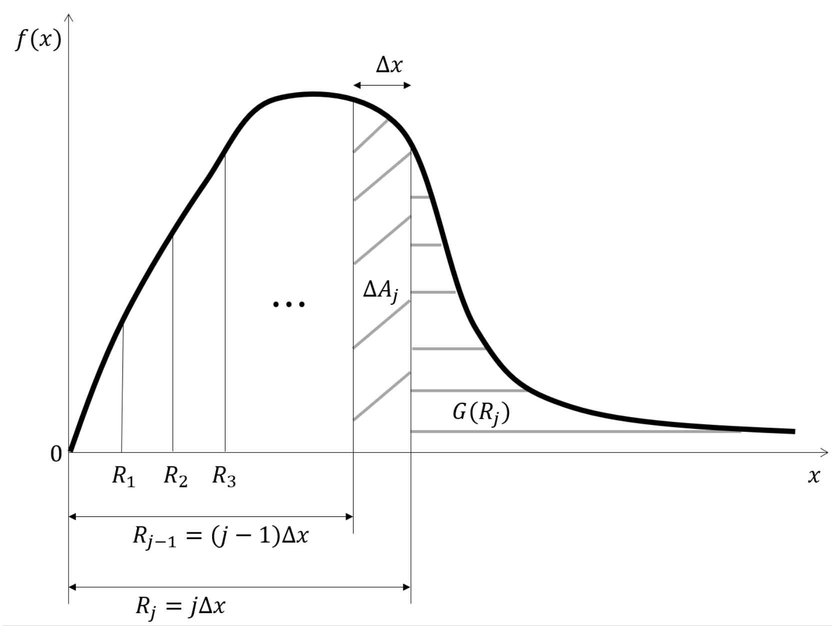
For Questions 8-13, recall the Probabilistic EOQ Model explained during lectures. The optimal values of y^* and R^* are given as follows

$$y^* = \sqrt{\frac{2D(K + pS(R^*))}{h}}$$

$$G(R^*) := \int_{R^*}^{\infty} f(x)dx = \frac{hy^*}{pD}$$

$$S(R^*) = \int_{R^*}^{\infty} (x - R^*) f(x)dx$$

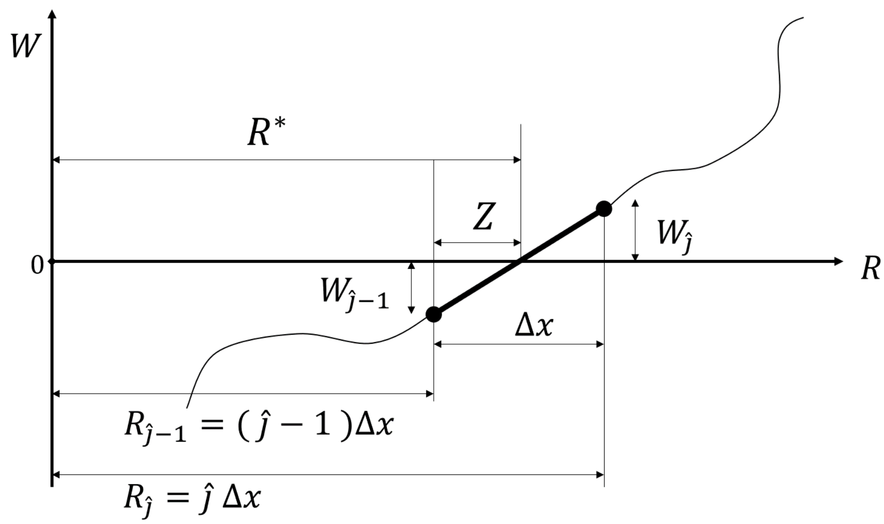
Here x is a random variable denoting the demand during lead time. Now assume that its PDF $f(x)$ is as shown below.



We discretize the $x-$axis with a *small enough* step size Δx. The figure also defines ΔA_j and the variables $R_j = j\Delta x$.

8) (0.25 mark) Determine the missing lower limit for the summation operator such that $G(R_j) = \sum_{k=?}^{\infty} \Delta A_k$
9) (0.25 mark) Show that $G(R_j) = G(R_{j-1}) - \Delta A_j$
10) (2 marks) Show that $S(R_j) = \Delta A_{j+1}\ (0.5\Delta x) + \Delta A_{j+2}\ (1.5\Delta x) + \cdots$

11) (1 mark) Show that $S(R_j) = S(R_{j-1}) - [G(R_j) + 0.5\Delta A_j]\Delta x$
12) (1 mark) Show that $[G(R^*)]^2 = c_1 + c_2 S(R^*)$ where c_1, c_2 are constants.
13) (2 marks) We want to compute R^* using the previous equation. Assume that a unique optimal value of R^* exists. Define $W(R) = c_1 + c_2 S(R) - [G(R)]^2$ and let W_j denote $W(R_j)$. Let W be plotted against R as shown below. Note that R^* is located where the graph crosses the $x-$axis. We can always approximate a curve to be a straight line within a small neighborhood. Assume that this unique zero of $W(R)$ lies between $R_{\hat{j}-1}$ and $R_{\hat{j}}$ and that you have found $\hat{j}$ by some means, then express R^* in terms of $\hat{j}$, $W_{\hat{j}-1}$, $W_{\hat{j}}$ and Δx.

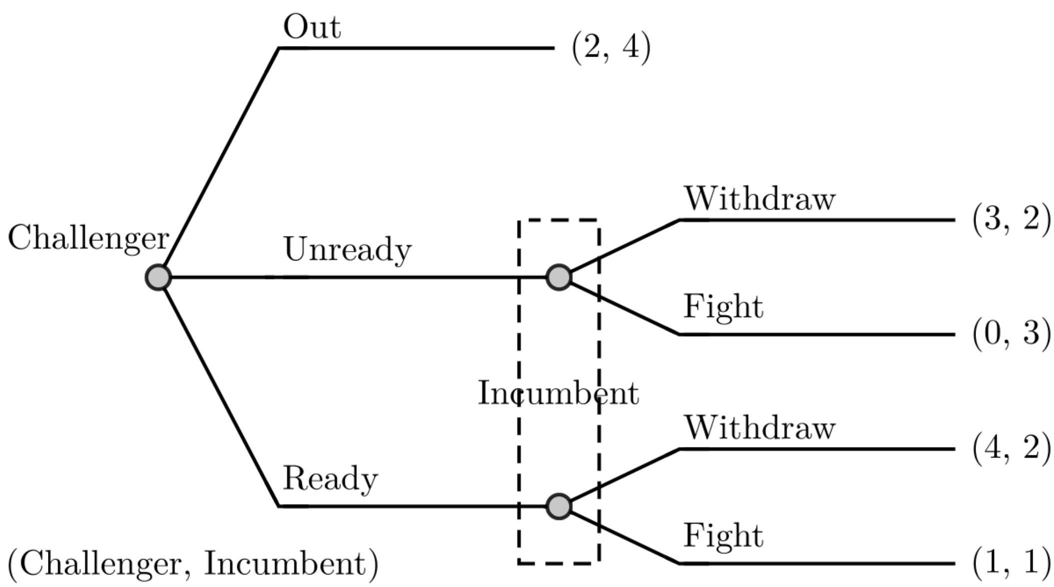


For Questions 14-17, you must make use of mathematical notation. A purely English justification shall be penalized.

14) (1 mark) Show that if X is a strongly dominant strategy for Player 1, then Player 1 uses X in any Nash equilibrium.
15) (1 mark) Show that if a strategy profile in a simultaneous strategic game for n players is composed of only the best response strategies, then that strategy profile is a Nash equilibrium.
16) (1 mark) Show that if the consistency equation for extensive games holds for all but one non-terminal history in an information set, then the consistency equation also holds for the last remaining non-terminal history.
17) (3 marks) Show that if X is a dominant strategy for Player 1, then X is also a prudential strategy for Player 1.

For Questions 18-23, consider two agents 1 and 2 who are deciding to build a firm. The firm's revenue depends on the amount of work done by each agent and is given by: $R(W_1, W_2) = c_1 W_1 + c_2 W_2$ where W_i is the amount of work done by agent $i = 1, 2$ and $c_1, c_2 > 0$ are constants. The first agent is contractually stipulated to receive fraction $0 \leq f \leq 1$ of the firm's revenue and remaining goes to the other agent. Each agent dislikes work and penalizes it using a cost term of W_i^2. Assume $W_i \geq 0$. Model this decision-making scenario as a strategic game.

18) (1 mark) What are the strategies available to each agent?
19) (1 mark) Define the agents' payoffs for each strategy profile assuming that it is the amount of revenue that the agent receives minus the work penalty.
20) (1 mark) Compute the strategy that agent 1 chooses using the dominance solution concept, meaning maximize agent 1's payoff assuming agent 2's amount of work can be treated as constant.
21) (1 mark) Compute the strategy that agent 2 chooses using the dominance solution concept, meaning maximize agent 2's payoff assuming agent 1's amount of work can be treated as constant.
22) (1 mark) Now compute the revenue of the firm assuming that both agents adopt their dominant strategy. For what value of f is the revenue maximized?
23) (1 mark) Also compute the difference between revenue of the firm and the agents' work penalties assuming that both agents adopt their dominant strategy. For what value of f is the difference maximized?
24) (5 marks) Find all weak sequential equilibria for the extensive game tree shown below. Proceed like the analysis of the war game shown during lectures. Fully indicate the supposed probabilities on the game tree. Explicitly state all equilibria by specifying their s and β and putting a box around each equilibrium.


Out
(2, 4)
Challenger
Unready
Withdraw
(3, 2)
Fight
(0, 3)
Incumbent
Withdraw
(4, 2)
Ready
Fight
(1, 1)
(Challenger, Incumbent)