# MCL 702: ADVANCED FLUID MECHANICS

## Minor Exam # 1

Date: September 9, 2017

Time: 1 hour Marks: 20

<u>*Instructions:*</u>

1. This exam is open book, open notes.
2. Show all necessary and important steps used to obtain your solution.

**PROBLEM 1 (9 marks)**

Prove the following identities using index notation:

a) $\nabla.\left(\nabla\times\vec{u}\right)=0$

b) $\nabla\cdot\left(\phi\vec{u}\right)=\phi\nabla\cdot\vec{u}+\vec{u}\cdot\nabla\phi$

c) $\nabla\times\left(\vec{u}\times\vec{v}\right)=\vec{v}.\nabla\vec{u}-\vec{u}.\nabla\vec{v}+\vec{u}\nabla.\vec{v}-\vec{v}\nabla.\vec{u}$

**PROBLEM 2 (4 marks)**

A two-dimensional unsteady field is given by $u=x\left(1+2t\right)$, $v=y$. Find the equation of the time-varying streamlines that all pass through the point $\left(x_0,y_0\right)$ at some time $t$.

**PROBLEM 3 (7 marks)**

For the two-dimensional linear shear flow $u=cy$, $v=0$,

a) Derive the components of the strain rate tensor $\left(S_{ij}\right)$ and vorticity vector.

b) Also derive the components of the rotational $\left(\frac{d\vec{v}^{(r)}}{ds}\right)$, straining $\left(\frac{d\vec{v}^{(s)}}{ds}\right)$, elongational $\left(\frac{d\vec{v}^{(es)}}{ds}\right)$ and shearing $\left(\frac{d\vec{v}^{(ss)}}{ds}\right)$ components of motion of a point P' at a distance $ds$ from P. Take the unit vector from P to P' be $\vec{\alpha}=\alpha_1\hat{i}+\alpha_2\hat{j}$.

c) Suggest the directions along which maximum and zero shearing deformations will occur.