MCL 702: ADVANCED FLUID MECHANICS

Minor Exam # 1

Date: August 25, 2018

Time: 1 hour Marks: 20

Instructions:

1. This exam is open book, open notes.
2. Show all necessary and important steps used to obtain your solution.

PROBLEM 1 (6 marks)

Prove the following identities using index notation:

a) $\nabla\cdot\left(\phi\vec{u}\right)=\phi\nabla\cdot\vec{u}+\vec{u}\cdot\nabla\phi$

b) $\nabla\times\left(\vec{u}\times\vec{v}\right)=\vec{v}.\nabla\vec{u}-\vec{u}.\nabla\vec{v}+\vec{u}\nabla.\vec{v}-\vec{v}\nabla.\vec{u}$

PROBLEM 2 (4 marks)

A two-dimensional unsteady field is given by $u=x\left(1+2t\right)$, $v=y$. Find the equation of the time-varying streamlines that all pass through the point $\left(x_0,y_0\right)$ at some time t.

PROBLEM 3 (10 marks)

For the two-dimensional flow between two parallel plates separated by a distance $2h$, the flow profile is given by $u=u_0\left[1-\left(\frac{y}{h}\right)^2\right]$, $v=0$. Here u_0 is the maximum velocity with the plates located at $y=\pm h$. Consider a point P that is located at the coordinate $y=h/2$.

a) Derive the components of the strain rate tensor $\left(S_{ij}\right)$ and vorticity vector at point P.

b) Consider two points Q and R located at angles of 45° and 90° and distance ds from P, respectively. Calculate the values of the components of elongation $\left(\frac{d\vec{v}^{(es)}}{ds}\right)$ and shearing strain $\left(\frac{d\vec{v}^{(ss)}}{ds}\right)$ motions for points Q and R for the given flow.