Department of Mechanical Engineering, IIT Delhi
Major: MCL 731 Analytical Dynamics

Instructors	Rama Krishna K, S.K. Saha	Marks	35
Venue	LH 316	Duration	13:00-15:00 (2 hours)
Date	Nov. 22, 2018 Thursday		

Instructions

- *Don't keep mobile with you. Keep in the front;*
- *Don't share calculator, Pencil, Compass, etc.*
- *Don't ask anything about the question paper (Do whatever you feel best!)*

Show your I-card when signing the attendance sheet

1. Answer the following: [5 × 3 = 15]

a) What is Euler-Bernouli beam?
b) Write the key steps to derive the equations of motion of vibrating string.
c) Define geometric theory, phase portrait, singular points, focus and center with sketches.
d) State Euler equations for rotation motion of a rigid body.
e) State Lyapunov's Direct Method for stability.

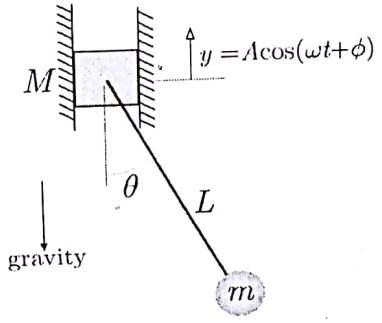
2. Considering the rotational speeds of a rigid body about Z-axis, new X-axis, and latest Z-axis as $\dot{\psi}$, $\dot{\theta}$, and $\dot{\phi}$, respectively, which can be denoted as $[0 \quad 0 \quad \dot{\psi}]^T$, $[0 \quad \dot{\theta} \quad 0]^T$, and $[0 \quad 0 \quad \dot{\phi}]^T$, in their local frames, derive the angular velocity of the rigid body in the fixed-frame by in terms of the rates of the ZXZ Euler angles, i.e., $[\dot{\psi} \quad \dot{\theta} \quad \dot{\phi}]^T$. [10]

3. For a given linear state-space system given by $\dot{\boldsymbol{x}} = \boldsymbol{Ax}$, if you choose a constant symmetric positive definite matrix $\mathbf{P}$ to define a Lyapunov function $V = x^T \boldsymbol{Px}$ then what is the condition for the criterion of Lyapunov stability. Illustrate with an example of a mass (60 kg)-spring (6000 N/m)-damper (600 Ns/m) system. Find a function V for asymptotic stability. [5]

4. In the figure shown below, the reference for zero potential energy is $y = 0$ and the position of mass M oscillating in a frictionless vertical guide is given by $y = A\cos(\omega t + \phi)$. Mass m is connected to M through a massless rigid rod of length L and a frictionless hinge. Using only θ as the generalized coordinate,

(a) Find the generalized momentum p_θ [2]

(b) Write the expression for the Hamiltonian function $H(\theta, p_\theta, t)$ by eliminating $\dot{\theta}$. [3]



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