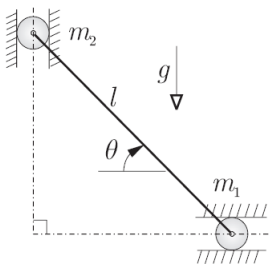
Department of Mechanical Engineering, IIT Delhi
Major: MCL 731 Analytical Dynamics

Instructors	Rama Krishna K, S.K. Saha	Marks	40
Venue	Online	Duration	2 hr
Date	Nov. 16, 2021		

Instructions

- *Open-book examination;*
- *Don't ask anything about the question paper (Do whatever you feel best!)*

1. Two particles having masses m_1 and m_2 connected by a massless rigid rod of length l are constrained to slide in two orthogonal frictionless grooves as shown in the figure below. Using only θ as the generalized coordinate, write the expression for the Hamiltonian. [5]



2. Answer the following: [5 x 3=15]

a) How could you study the stability of a first-order dynamical system?
b) What are the properties of a rotation matrix? Illustrate using a rotation of a body about Y-axis.
c) What is geometric theory? How is it used to study equilibrium of a dynamical system?
d) What is Lyapunov's direct method for stability criterion?
e) What is Routh-Herwitz criterion? What is it used for?

3. Write the equations of motion of a planar solid pendulum of length a (slender rod) and mass m using Newton's 2nd law for translational motion and Euler's equations for rotational motion. [Hint: Use free-body diagram indicating reaction forces after removing the hinge joint of the solid pendulum.] [6]

4. Considering the rotational speeds of a rigid body about Z-axis, new X-axis, and latest Z-axis as $\dot{\psi}$, $\dot{\theta}$, and $\dot{\phi}$, respectively, which can be denoted as $[0 \quad 0 \quad \dot{\psi}]^T$, $[\dot{\theta} \quad 0 \quad 0]^T$, and $[0 \quad 0 \quad \dot{\phi}]^T$, in their local frames, derive the angular velocity of the rigid body in the fixed-frame by in terms of the rates of the ZXZ Euler angles, i.e., $[\dot{\psi} \quad \dot{\theta} \quad \dot{\phi}]^T$. [6]

5. Derive the equations of motion of a vibrating string using Hamilton's principle. [8]

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