Venue: Home | Odd Semester 2021-2022 | 20 September 2021

Department of Mechanical Engineering
Indian Institute of Technology Delhi
MCL731: Analytical Dynamics

Time: 9.00 am-10.45 am | Minor Test | Maximum Marks: 30

Instructions

Use of any electronic devices not permitted; *Follow academic honor code*; *Assume appropriately any missing data*

1. Derive Lagrange's equation of motion of the second kind from D'Alembert's principle for a holonomic system without constraints [04]
2. Consider a simple pendulum of mass m and length l constrained to move in the xy plane. Gravity acts along negative y-axis. Is the system conservative or non-conservative? Give reasons. [01]
3. Explain why obtaining the equations of motion (in Cartesian coordinates) of a spherical pendulum using D'Alembert's principle together with the constraint equation is relatively easier than using Lagrange's equations of motion of the first-kind together with the same constraint equation. [01]
4. Given a physical system with a Lagrangian function $L = \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2+2\beta z)$ and a constraint $a\dot{x} + b\dot{y} + c\dot{z} = 0$ where x, y, and z are the generalized coordinates and β, a, b and c are some non-zero constants.
   (a) Solve for $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$ in terms of the symbols introduced above. [04]
   (b) Solve for the constraint forces. [03]
5. Referring to Fig. 1, write the nonholonomic constraint equation/s involving the generalized coordinates x, y, θ and ϕ. Angle θ is the angle between the disk axis and x-axis. [03]

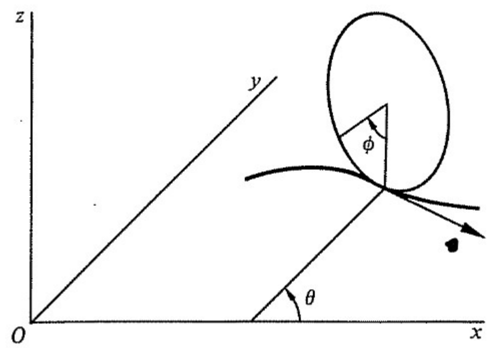

Fig. 1: Vertical rolling disk on a horizontal plane.

6. A particle is constrained to move along a equiangular spiral $r = ae^{b\theta}$ so that the radius vector moves with constant angular velocity ω. Determine the velocity and acceleration components in terms of a, b, ω, and t. [04]

7. In Fig. 2, two particles each of mass of m are connected by a rigid massless rod of length l. Particle 1 can slide without friction on a fixed straight wire. Using (x, θ) as generalized coordinates, find expressions for the kinetic energy and the generalized momentum p_θ [04]

8. A particle of mass m can slide without friction in a straight slot cut in a horizontal turntable (Fig. 3). The turntable rotates at a constant angular velocity Ω about a vertical axis through its center at O. The coordinate y represents the position of the particle relative to the turntable and is equal to zero when the spring is unstressed and the particle is at minimum distance R from the center O. Use the Lagrangian method to the differential equation of motion. Identify the terms generated by T_2, T_1, T_0, and V. [06]

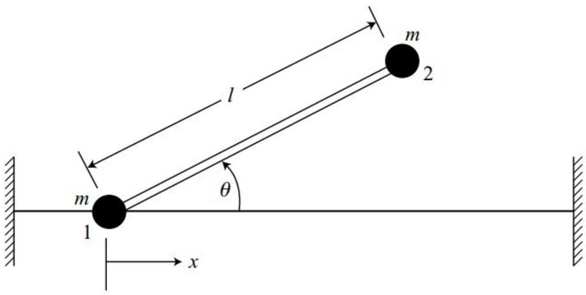

Fig. 2: Constrained dumbell.

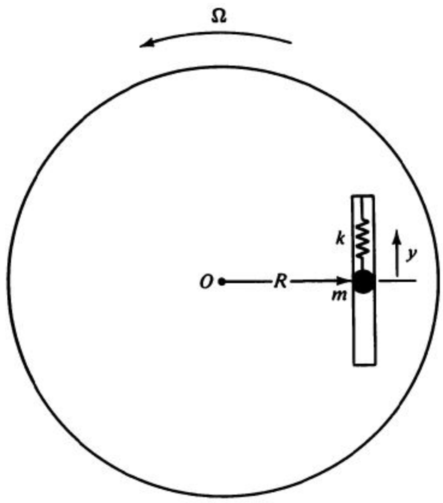

Fig. 3: Mass in a groove.