Venue: LHC 308 | Odd Semester 2024-2025 | 13 September 2024

Department of Mechanical Engineering
Indian Institute of Technology Delhi
MCL731: Analytical Dynamics

Time: 15:30-17:30 | Minor Examination | Maximum Marks: 30

Instructions

Use of any electronic devices (except calculator) not permitted; Follow academic honor code; Assume appropriately any missing data

1. Write Lagrange's equations of motion of the second kind for a system having n generalized coordinates, m holonomic constraints, and l non-holonomic constraints. [02]

2. State the Principle of Virtual Work. [02]

3. By means of a rack-and-pinion mechanism, large forces can be developed by the cork puller shown in Fig. 1. If the mean radius of the pinion gears is 12 mm, determine the force R which is exerted on the cork for given forces P on the handles. Solve the problem using the principle of virtual work. Assume that the negative work of friction is negligible. [04]

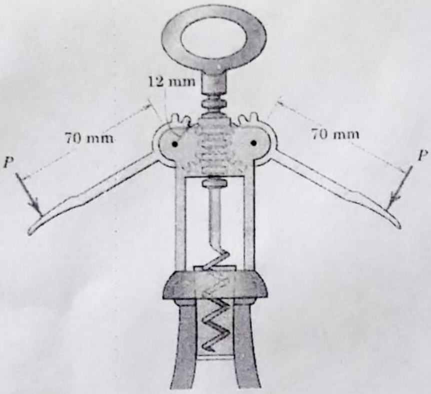

Fig. 1: Wing-style wine bottle cork puller.

4. A particle moves in a plane with constant radial velocity $\dot{r} = 4$ m/s, starting from the origin. The angular velocity is constant and has magnitude $\dot{\theta} = 2$ rad/s. When the particle is 3 m from the origin, find the magnitude of:
   (a) velocity [02]
   (b) acceleration [03]

5. Prove the "Dot Removal Lemma" given by: $\dfrac{\partial \dot{\boldsymbol{r}}_i}{\partial \dot{q}_j} = \dfrac{\partial \boldsymbol{r}_i}{\partial q_j}$ [02]

6. Show that $\dfrac{d}{dt}\left(\dfrac{\partial \boldsymbol{r}_i}{\partial q_j}\right) = \dfrac{\partial \dot{\boldsymbol{r}}_i}{\partial q_j}$ [02]

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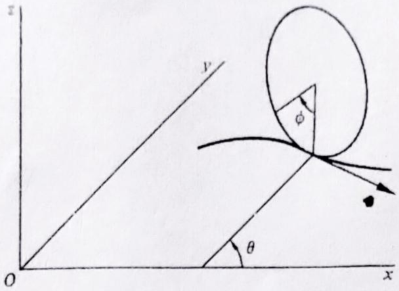

Fig. 2: Vertical rolling disk on a horizontal plane.

7. Referring to Fig. 2, write the non-holonomic constraint equation/s involving the generalized coordinates x, y, θ and ϕ. Angle θ is the angle between the disk axis and x-axis. [03]

8. In Fig. 3, two particles each of mass of m are connected by a rigid massless rod of length l in vertical plane under gravity. Particle 1 can slide without friction on a fixed straight wire. (x, θ) are the generalized coordinates.
   - (a) Find expressions for the kinetic energy in terms of the generalized coordinates. [02]
   - (b) Determine the Lagrangian of the system. [02]
   - (c) Find the generalized momentum p_θ. [02]
   - (d) Derive the equations of motion using the standard Lagrange's equations of motion of the second kind. [04]

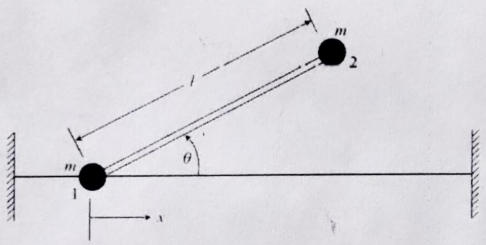

Fig. 3: Constrained dumbell.

The reader will find no figures in this work. The methods which I set forth do not require either constructions or geometrical or mechanical reasonings: but only algebraic operations, subject to a regular and uniform rule of procedure.

\- J. L. Lagrange in Méchaninique Analytique (1788)