Venue: LHC 316 | Odd Semester 2018-2019 | 4 October 2018

Department of Mechanical Engineering
Indian Institute of Technology Delhi
MCL731: Analytical Dynamics

Time: 9.30 am-10.30 am | Minor Test - II | Maximum Marks: 20

Instructions

Use of any electronic devices not permitted; Do not share pencil,eraser,ruler; Assume appropriately any missing data

1. (a) If a coordinate frame is rotated about its Z-axis by an angle ψ, followed by a rotation of angle θ about the new Y-axis, then what is the final orientation matrix? Show graphically the final orientation of the coordinate frame, if $\psi = 90°$ and $\theta = 30°$. What is the final orientation matrix? [03]

   (b) If only rotation about Y-axis is imparted, prove that the angular velocity matrix, Ω_θ, is given by

$$\Omega_\theta = \begin{bmatrix} 0 & 0 & \dot{\theta} \\ 0 & 0 & 0 \\ -\dot{\theta} & 0 & 0 \end{bmatrix}$$

   where $\dot{\theta}$ is the time rate of change of angle θ. What is the corresponding angular velocity vector? [02]

2. Given a physical system with a Lagrangian function $L = \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2+2\beta z)$ and a constraint $a\dot{x} + b\dot{y} + c\dot{z} = 0$ where x, y, and z are the generalized coordinates and β, a, b and c are some non-zero constants.

   (a) Solve for $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$ in terms of the symbols introduced above. [03]

   (b) Solve for the constraint forces. [02]

3. Derive Lagrange's equations of motion of the second-kind from Hamilton's principle using the functional I given by

$$I = \int_{t_1}^{t_2} \left\{ L(q_1, ..., q_n, \dot{q}_1, ..., \dot{q}_n, t) - \sum_{j=1}^{m} \lambda_j \phi_j \right\} dt$$

   for a holonomic system subjected to m constraints of the form $\phi_j(q_1, ..., q_n, t) = 0$ for $j = 1, 2, ..., m$. [05]

4. The Lagrangian function of an unconstrained physical system can be written in the form

$$L = a(\dot{z} + \dot{y}\cos x)^2 + b(\dot{x}^2 + \dot{y}^2 \sin^2 x) + c\cos x$$

   where x, y, and z are the generalized coordinates and a, b, and c are some non-zero constants. Find the Routhian function. [03]

5. Is the system in Question 2 conservative or non-conservative? Give reasons. [02]