Minor 2, MCL 742 Design and Optimization — When: 6^{th} Oct 2017, 2:30-3:30 PM

Dept. of Mechanical Engineering, IIT Delhi — Points: 60

Note: Answer all questions. Calculator is permitted. No partial credit where there are multiple answers.

1. [10 points: **-1 for every wrong answer**] State True/False.
   a) Maximization of $f(x)$ is equivalent to minimization of $1/f(x)$.
   b) If a constant is added to a function, the location of its minimum point is changed.
   c) If a function is multiplied by a positive constant, the location of the function's minimum point is changed.
   d) The Hessian of an unconstrained function at its local minimum point must be positive semidefinite.
   e) In optimization formulation, number of inequality constraints cannot exceed number of design variables.
   f) Optimum design points having at least one active constraints give stationary value to the cost function.
   g) In unconstrained optimization, the cost function can increase for a small step along the descent direction.
   h) After initial bracketing, the golden section search required two function evaluations to reduce the interval of uncertainty.
   i) In minimization problem, for a "≥ type" constraint the sign of Lagrange multiplier is always negative.
   j) If a slack variable has zero value at the optimum, the inequality constraint is inactive.
2. [5 points] Match the following equations and their characteristics:

a) $f = 4x_1 - 3x_2 + 2$	Relative maximum at (1,2)
b) $f = (2x_1 - 2)^2 + (x_2 - 2)^2$	Saddle point at origin
c) $f = -(x_1 - 1)^2 - (x_2 - 2)^2$	No minimum
d) $f = x_1 x_2$	Inflection point at origin
e) $f = x^3$	Relative minimum at (1,2)

3. [6 points] Answer whether the following functions are (i) positive definite (ii) semi-positive definite (iii) negative definite (iv) semi-negative definite (v) null definite (vi) indefinite.
   a) $f = {x_1}^2 - {x_2}^2$
   b) $f = -{x_1}^2 + 4x_1x_2 + {x_2}^2$
4. [4 points] According to Wierstraas theorem, what can be said about the existence of global minimum?

   minimize $f = x_1 + x_2$
   subject to $x_1 x_2 \geq 1, x_1 \geq 0, x_2 \geq 0$
5. [14 points] A rectangular box of height a and width b is placed adjacent to a wall. Find the length of the shortest ladder that can be made to lean against the wall.
6. [21 points] A manufacturing firm producing small refrigerators has entered into a contract to supply 50 refrigerators at the end of first month, 50 at the end of the second month, and 50 at the end of the third. The cost of producing x refrigerators in any month is given by $(x^2 + 1000)$. The firm can produce more refrigerators in any month and can carry them to a subsequent month. However, it costs \$20 per unit for any refrigerator carried over from one month to the next. Assuming that there is no initial inventory, determine the number of refrigerators to be procured in each month to minimize the total cost. Use alternative formulation.