MCL765 Operations Research: Majors

November 24, 2021; 9.30-11.30AM (online) **Maximum Marks: 35 marks**

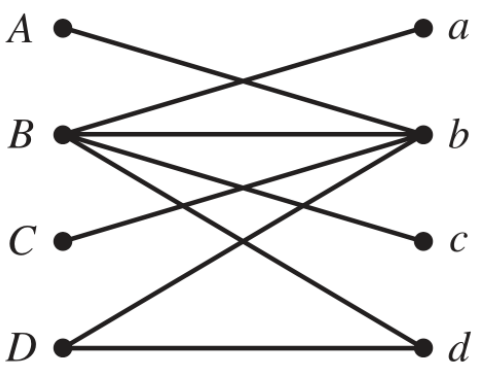
Q1) [4 marks] Consider the following linear-programming system of difference constraints.

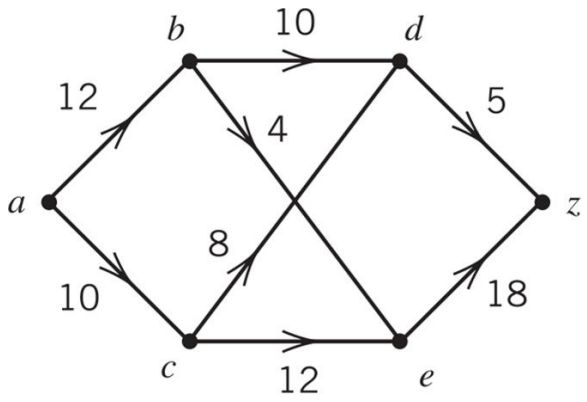
$$\begin{array}{lll} x_1 - x_4 \leq -1 & x_2 - x_3 = -9 & x_4 - x_3 \leq -3 \\ x_1 - x_5 \leq -4 & x_3 - x_1 \leq 5 & x_5 - x_1 \leq 5 \\ x_2 - x_1 \leq -4 & x_3 - x_5 \leq 2 & x_5 - x_4 \leq 1 \end{array}$$

Show the constraint graph for these 9 constraints following the convention explained during the lectures.

Q2) [3 marks] Can you solve the constraint graph obtained above?

Q3) [1 mark] Why perfect matching will not be possible for the bipartite graph shown below on the left?





Q4) [7+3 marks] Find a maximum $a-z$ flow and minimum capacity $a-z$ cut in the network shown above on the right. Make sure to show the flows in the given network and weights in the corresponding residual network after each iteration as shown during lectures. Also identify the augmenting paths.

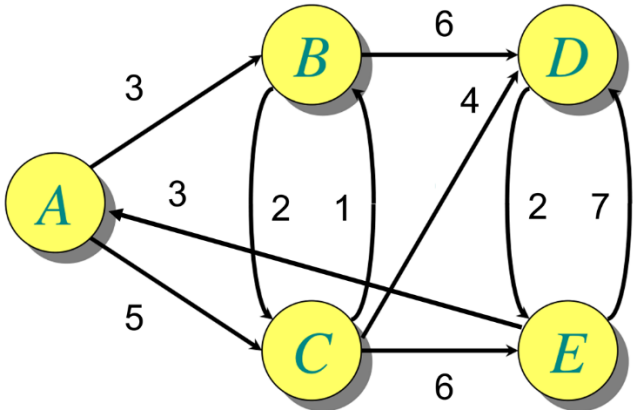
Q5) [1 mark] If a dynamic-programming problem satisfies the optimal-substructure property, then a locally optimal solution is globally optimal. True or False?

Q6) [1 mark] In any bipartite graph, the number of edges in a minimum matching equals the number of vertices in a maximum vertex cover. True or False?

Q7) [2 marks] Why are there an even number of odd-degree vertices in a graph?

Q8) [2 marks] What is the cardinality of the largest independent set of edges in a cycle of length 7?

Q9) [4 marks] Run Dijkstra's algorithm on the directed graph shown below, using vertex A as the source. Make sure to show the d and π values and the vertices in set S after each iteration as shown during lectures.



Q10) [4 marks] Run Dijkstra's algorithm on the directed graph shown above, using vertex E as the source. Make sure to show the d and π values and the vertices in set S after each iteration as shown during lectures.

Q11) [1 mark] Minimize the number of vertices a graph can have if it has 50 edges.

Q12) [2 marks] Show a graph that contains a Eulerian tour but has no Hamiltonian tour.