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MCL765 Major Exam (22 Nov 2022)

Avoid skipping steps. Cheating shall be heavily penalized. Max. marks: 30

1. (3 marks) Consider a discrete random variable X having n possible outcomes. which takes value x_i with probability p_i. Using LMM. maximize $H(X) = -\sum_i p_i \log p_i$ subject to the normalization condition.

2. (2 marks) Using LMM, show that the optimal solution for the problem of maximizing $f(x_1, x_2) = \rho_1 \log x_1 + \rho_2 \log x_2$ subject to the constraint $\sum x_i = X$ is given by

$$x_i^* = \frac{X\rho_i}{\sum \rho_i}$$

3. (2 marks) If a, b, c, and d are positive numbers, using AM-GM inequality, then find a lower bound on

$$(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

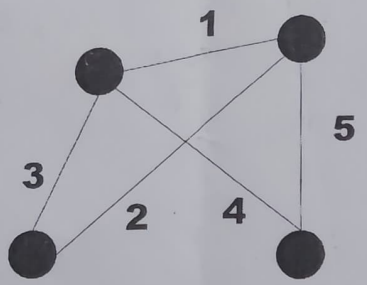
4. (2 marks) Using AM-GM inequality, prove that if $x \geq 0$, then $3x^3 - 6x^2 + 4 \geq 0$.

5. (2 marks) If the sum of three positive numbers is six, then minimize the sum of their squares.

6. (5 marks) Consider maximizing $F_0(y) = \int_0^a y(x)\,dx$ subject to the constraint $F_1(y) = \int_0^a \sqrt{1+(y')^2}\,dx = \ell$. Find the required differential equation using calculus of variation.

7. (1 mark) If all the edge weights of a connected graph are distinct, then how many MSTs are possible in the graph?

8. (3 marks) Find the MSTs of the graph given below. Then find two second-best minimum spanning trees for the graph given below. Give the formal definition of the second-best minimum spanning tree used by you to solve this problem.



9. (3 marks) Write down the pseudocode of Prim's algorithm and then perform its time complexity analysis assuming the usage of binary heap data structure.

10. (1 mark) How are Prim's algorithm and Dijkstra's algorithm different? Explain.

11. (3 marks) Consider the system of difference constraints given below. Find a feasible solution for this system by using a graph-theoretical method.

$$\begin{aligned} x_1 - x_2 &\leq 5 \\ x_1 - x_3 &\leq 6 \\ x_2 - x_4 &\leq -1 \\ x_3 - x_4 &\leq -2 \\ x_4 - x_1 &\leq -3 \end{aligned}$$

12. (1 mark) Does the previous system of difference constraints have a unique solution? Why?