DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY PUNJAB (IIT DELHI CAMPUS)

MAJOR TEST 2008-09 SECOND SEMESTER

MAL 120 (MATHEMATICS-II)

Time: 2 hours **Max. Marks: 50**

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1. Let f(z) be analytic in the whole complex plane such that for all $r > 0$

$$\int_0^{2\pi} | f(re^{i\theta}) | \, d\theta \leq \sqrt{r},$$

Find $\frac{f^{(n)}(0)}{n!}$ for all $n \geq 0$. (3)

2. Let α be any complex number and $\overline{\alpha}$ be its complex conjugate. For the transformation $w = \frac{z-\alpha}{z-\overline{\alpha}}$, find the region in the W-plane which corresponds to the interior of a circle with centre $\overline{\alpha}$ and radius $\rho > 0$ in the Z-plane. (3)

3. Using the method of residues evaluate the integral

$$\int_0^{2\pi} \frac{d\theta}{(5 - 3\sin\theta)^2}.$$

(4)

4. Find all Laurent series expansions of the function $f(z) = \frac{3z+6}{z^2+3z}$ with centre at z=3. Also specify their respective regions of validity. (6)

5. Using Fourier sine integral of $f(x) = e^{-x}\cos x, \quad x > 0$ compute

$$\int_0^{\infty} \frac{\omega^3 \sin \omega x}{\omega^4 + 4} \, d\omega$$

(5)

6. Prove that

$$\int_{(1,0)}^{(2,1)} [(2xy - y^4 + 3)\, dx + (x^2 - 4xy^3)\, dy]$$

is independent of the path joining (1,0) and (2,1) and evaluate the integral. (4)

7. Change the order of integration in

$$I = \int_0^1 \int_{1-\sqrt{1-y}}^{1+\sqrt{1-y}} \frac{1}{(x^2 - 2x + y - 3)^2} \, dx dy$$

and hence evaluate the same. (4)

P.T.O.

8a. Find the surface area of the surface which is the cut from the paraboloid $x^2 + y + z^2 = 2$ by the plane $y = 0$. (4)

8b. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes y+z=4 and z=0. (4)

9a. Let $f(x, y, z)$ be a scalar function which satisfies $\nabla^2 f = 0$. Evaluate

$$\int\int_S \text{grad } f.\hat{n}\, ds,$$

where $\hat{n}$ is the unit outward normal vector to the surface S and S encloses a volume V. (3)

9b. Let $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, S be the surface of the sphere $x^2 + y^2 + z^2 = 1$ and $\hat{n}$ be the inward unit normal vector to S. Find the value of

$$\int\int_S \vec{F}.\hat{n}\, ds$$

(2)

10a. State Green's theorem in a plane. (2)

10b. Prove or disprove that Green's theorem in a plane is a special case of Stokes theorem. (3)

10c. Suppose that the function $w(x, y)$ satisfies $\nabla^2 w = 0$. Then find the value of $\int_C \text{grad } w.\vec{n}\, ds$, where $\vec{n}$ is the unit normal to the curve C and s is its arc length. (3)