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Department of Mathematics
MAL120 Mathematics II
Major: Semester II (2013 - 14)

Max Time: 2 hrs

Total marks: 50

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1. Find the volume of the region in the first octant bounded by the coordinate planes and the planes $x + z = 1$, $y + 2z = 2$. (4)

2. Find the surface area of the cylinder $x^2+y^2 = 4$ cut by the cylinder $y^2+z^2 = 4$. (6)

3. Evaluate $\iint_S (\nabla \times \vec{F}).\hat{n}\, dS$, where $\vec{F} = (x - z)\vec{i} + (x^3 + yz)\vec{j} - 3x^2y\vec{k}$ and S is the surface of the cone $z = 2 - \sqrt{x^2 + y^2}$, $z \geq 0$. (6)

4. (a) Suppose that $f(z)$ is analytic inside and on a simple closed curve C. Show that (4)

$$\int_C \overline{f(z)} f'(z) dz$$

is purely imaginary.

(b) Evaluate (3)

$$\oint_{|z|=1} e^{1/z^n}\, dz, \quad n \in \mathbb{N}.$$

5. (a) Determine the radius of convergence of the power series (4)

$$\sum_{n=1}^{\infty} (-1)^n \frac{n! z^{2n}}{n^n}.$$

(b) Find the Laurent Series expansion of $f(z) = \frac{z^2-2z+5}{(z^2+1)(z-2)}$ in the domain $1 < |z| < 2$ with center $z = 0$. (4)

6. Classify the nature of the isolated singularities of the following functions and determine the residue at each singularity (with explanation):
(i) $\frac{z}{\sin(\pi/z)}$ (ii) $\frac{\tan z}{1-e^z}$ (6)

7. Let C_R be the portion of the circle $|z| = R$ with $\mathrm{Im}(z) \geq 0$. Then show that (6)

$$\lim_{R\to\infty} \left| \int_{C_R} \frac{(z+1)e^{iz}}{z^2+4z+5} dz \right| = 0.$$

8. Using residue theorem evaluate $\int_0^{\infty} \frac{\sin x}{x}\, dx$. (7)