DEPARTMENT OF MATHEMATICS
MTL 100: Calculus

Major - Nov. 2015 *Marks - 40*

Answer as is asked. All questions are compulsory.

(1.) (a) Find $\limsup s_n$ and $\liminf s_n$, when
(i) $s_n = \frac{3}{4n+1}$,
(ii) $s_n = (-1)^n + \frac{1}{n}$. [2 marks]
(b) Let f, g be differentiable functions on $\mathbb{R}$. If $f(0) = g(0)$ and if $f'(x) \le g'(x)$ then show that $f(x) \le g(x)$ for all $x \ge 0$. [3 marks]

(2.) (a) Determine whether $\sum\limits_{n=1}^{\infty} \frac{e^n}{e^{2n}+1}$ converges. [3 marks]
(b) Write the Taylor series of $f(x) = x^2 - 8$ centered at $x_0 = 1$. [2 marks]

(3.) (a) Find the directional derivative of $f(x, y) = y\sin(xy)$ in the direction of $2i + j$ at $(\pi/8, 2)$. [3 marks]
(b) For the function $f(x, y) = x^2y - 2x^2 - y^2$ find the critical points and categorize each critical point as a relative maximum, relative minimum or saddle point. [3 marks]

(4.) (a) If $z = f(x, y)$, $x = e^u - e^v$, $y = e^{-u} + e^v$, then determine

$$\frac{\partial f}{\partial u} - \frac{\partial f}{\partial v} - x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$$

[3 marks]
(b) Find the area of the domain in the XY-plane bounded by the curves $x = y^2$, and $y = x^2$. [3 marks]

(5.) (a) Write down the iterated triple integral in cylindrical coordinates WITH PROPER LIMITS for the volume of D, the solid inside $x^2 + y^2 + z^2 = 9$ and above the plane $z = \sqrt{5}$. DO NOT EVALUATE. [3 marks]
(b) Let R be the region in the xy-plane bounded by $y = x^2$, $y = 4 - x^2$ and $y = 3x$. Let D be the solid above R and below the plane $x + y + z = 5$.
Set up an iterated integral for the volume of D. DO NOT EVALUATE. & for first quadrant [3 marks]

(6.) (a) Perform a change of variables and evaluate the integral $\iint_R x \, dA$ over a rectangle S in the uv-plane. Here R is the region bounded by the lines $y - x = 0, y - x = 2, 3x + y = 0$ and $3x + y = 4$. Make sure that all your steps are clear and draw both your regions R and S. [3 marks]
(b) Evaluate $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} \, dydx$. [3 marks]

(7.) Determine where the vector field F is conservative and compute $\int_C F \cdot dr$ for
(a) $F(x, y) = (2x - 6y + 7)i - (5x + 7y - 8)j$, and
(b) $F(x, y) = (3x^2y^2)i + (2x^3y)j$
where C is the curve first along the y-axis from $(0, 1)$ to $(0, -1)$ and then along the semi-circle $x^2 + y^2 = 1$ joining $(0, -1)$ to $(0, 1)$. [3+3 marks]