ANKIT KUMAR SINGH 2017CS10328

Department of Mathematics
MTL 100: Calculus
Major: 2017-18 Semester II

Max Time: 2 hrs

Total marks: 40

1. Use the Lagrange multipliers to find the maximum volume of a rectangular BOX having a surface area (of all six faces) 60 square inches. [5]

2. Consider the improper integral [3+2]
$$\int_0^\infty x^p e^{-x} dx.$$
   (a) Find the values of p (by giving a proof) for which the improper integral converges.
   (b) Find the values of p for which the improper integral diverges.

3. (a) Find the arc length of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$. [2+3]
   (b) Let $f : [0,1] \to \mathbb{R}$ be a continuous function. Prove that f is Riemann integrable.

4. Find the volume of the piece of the cylinder $x^2 + y^2 = 1$ that lies between the surfaces $z = x^2 + y^2$ and $z = 4 - x^2 - y^2$. [5]

5. Find the surface area of sphere $x^2 + y^2 + z^2 = 1$ lying between the cones $z = \sqrt{x^2 + y^2}$ and $z = 2\sqrt{x^2 + y^2}$. [5]

6. Evaluate using appropriate transformation
$$\iint_R \sqrt{x+y}\, dxdy,$$
where R is the parallelogram with vertices $(1,0), (3,1), (2,2)$ and $(0,1)$. [5]

7. Consider the function $f(x,y) = \begin{cases} \dfrac{|x|^{\frac{3}{2}} y}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$
   (a) Does the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exists? Justify.
   (b) Find the partial derivatives of f at $(0,0)$. [3+2]

8. (a) Prove that if a sequence $\{a_n\}$ is Cauchy then $\{a_n^2\}$ is also Cauchy sequence.
   (b) Test the absolute convergence of the series $\sum_{n=2}^{\infty} \dfrac{\sin(1/n)}{n \log n}$. [2+3]

—ALL THE BEST—