Department of Mathematics
Indian Institute of Technology Delhi
MTL 100 Calculus: Major

Total marks: 40

Time: 2 hours

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1. Every question is compulsory
2. All questions carry equal marks : $8 \times 5 = 40$
3. No marks will be provided if appropriate justification is not provided

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1. Discuss the convergence of the following sequences:

   (a) $\{\sqrt{n+1} - \sqrt{n}\}_{n=1}^{\infty}$ (b) $\{1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\}_{n=1}^{\infty}$. [2+3]

2. Discuss the convergence of the following series:

   (a) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ (b) $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ [2+3]

3. Using Lagrange multipliers find the maximum volume of a rectangular solid in the first octant with one vertex on the origin and the other vertex on the plane $x + y + \frac{z}{2} = 1$.

4. Show that a continuous function defined on a closed and bounded interval is Riemann integrable.

5. Evaluate the double integral $\int_0^3 \int_{x^2}^9 x^3 e^{y^3} \, dy dx$ by changing the order of integration.

6. Use the transformations $u = xy$ and $v = x^2 - y^2$ to change and evaluate the integral $\iint_R (x^2 + y^2)^3 \, dA$, where $R = \{(x, y) \in \mathbb{R}^2 : 1 \leq xy \leq 2, 1 \leq x^2 - y^2 \leq 2\}$.

7. Find the surface area of the surface $z = 1 - x^2 - y^2$above the xy-plane.

8. Find the volume of the solid obtained by cutting the cylinder $x^2 + y^2 = 4x$ by the surfaces $z = 0$ and $z = 8 - x^2 - y^2$.