Department of Mathematics
Indian Institute of Technology Delhi

# MTL 100 Calculus: Major

Time: 2 hours

Total marks: 45

1. Every question is compulsory
2. No marks will be provided if appropriate justification is not provided

1. Find the volume of the wedge cut from the cylinder $x^2 + y^2 = 1$ by the planes $y + z = 1$ and $z = 0$. [5]

2. Find the area of the region cut from the first quadrant by the cardioid $r = 1 + sin\theta$. [5]

3. Use the transformations $u = (2x - y)/2$, $v = y/2$ and $w = z/3$ to evaluate the integral [6]

$$\int_0^3 \int_0^4 \int_{y/2}^{(y/2)+1} \left(\frac{2x - y}{2} + \frac{z}{3}\right) dx\, dy\, dz.$$

4. Discuss the convergence of the following integrals: [3+3]

   (a) $\int\limits_1^\infty \frac{dx}{x^2 + \sqrt{x}}$ (b) $\int\limits_1^2 \frac{\sqrt{x}}{\ln x}\, dx.$

5. Let $f : [a, b] \to \mathbb{R}$ be a bounded function. Show that $f$ is integrable if and only if for every given $\epsilon > 0$ there exists a partition $P_\epsilon$ such that [5]

$$U(P_\epsilon; f) - L(P_\epsilon; f) < \epsilon.$$

6. Let $C$ be the ellipse obtained by the intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 1$. Using Lagrange multipliers, find the points on $C$ that lies closest and farthest from the origin. [6]

7. Find the critical points and their nature for the function

$$f(x, y) = 2x^3 + 2y^3 - 9x^2 + 3y^2 - 12y.$$

8. Determine (with justification) whether the following statements are true or false. [6]

(a) If $f$ is differentiable then $f'$ is continuous.

(b) Every uniformly continuous function is continuous.

(c) Every bounded sequence converges.