

Department of Mathematics
MAL120 - Mathematics II
Minor - 1: Semester II (2013-14)

Max Time: 1 hour

Total marks: 25

1. (a) Find the area of the region which lies inside the lemniscate $r^2 = \frac{2}{3} \sin 2\theta$ and outside the circle $r = 1$. (5)

(b) Find the volume of the region common to the interiors of the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$. (4)

2. Evaluate the following integral

$$\iint_R \cos\left(\frac{x-y}{x+y}\right) dx dy$$

where R is the region bounded by $x + y = 1$, $x = 0$, $y = 0$. (4)

3. Prove that $\int_{(1,1)}^{(2,3)} (2x^3y^4 + x) dx + (2x^4y^3 + y) dy$ is independent of the path joining $(1, 1)$ and $(2, 3)$, and find the integral value. (3)

4. Evaluate the line integral $\int_C (e^{x+y}x + x^2 \sin^2 x) dx + (e^{x+y}y + y^3 \cos y^2) dy$ where C denotes the boundary of the rectangle with vertices $(0, 0)$, $(1, 0)$, $(1, \frac{\pi}{2})$, $(0, \frac{\pi}{2})$ in the positive orientation. (4)

5. Find the surface area of a hemisphere of radius a cut off by a cylinder $x^2 + y^2 = ax$. (5)