a_1 $-a_2$ a_3 $-a_4$

Department of Mathematics
MTL 100: Calculus
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Minor 1

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$S_2 - S_1 = a_1 - a_2 - \ldots$

Please begin the answer to each question on a new page, and give adequate explanation for full credit.

1. Let A and B be two nonempty sets of real numbers such that $A \bigcup B = (0,1)$. Prove that this implies $\left(\inf A\right) \cdot \left(\inf B\right) = 0$, or give an example of sets A and B for which $\left(\inf A\right) \cdot \left(\inf B\right) > 0$. [4]

2. If $\left|a_n\right| < 2$ and $\left|a_{n+2} - a_{n+1}\right| \le \frac{1}{8}\left|a_{n+1}^2 - a_n^2\right|$ for $n \ge 1$, prove that the sequence $\left\{a_n\right\}$ converges. [4]

3. Discuss the convergence for the following two series:

   (a) $\displaystyle\sum_{n=1}^{\infty} n^{-1-1/n}$.

   (b) $\displaystyle\sum_{n=2}^{\infty} \left(\ln n\right)^p$, where $p < 0$.

   [2+2]

4. Prove that an alternating series $\sum(-1)^{n-1}a_n$ converges if $\left\{a_n\right\}$ is a nonincreasing sequence of positive real numbers that tends to 0. Further prove that if the series converges to S, then

$$\left|S - S_n\right| < a_{n+1}$$

for each $n \ge 1$ and $S_n = \displaystyle\sum_{i=1}^{n}(-1)^{i-1}a_i$. [4]

5. Let $f : [0,1] \to [0,1]$ be given by

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational;} \\ 1 - x & \text{if } x \text{ is irrational.} \end{cases}$$

Discuss the continuity of f in $[0,1]$ giving an $\epsilon - \delta$ argument. [4]