

Department of Mathematics

MTL 100: Calculus
Minor 1: 2017-18 Semester I

Total marks: 20
Max Time: 1 hr

1. (a) Prove that every bounded increasing sequence is convergent. [3+2]

(b) Let $a_n = \frac{1 + (-1)^n}{n^{\frac{1}{n}}}$, $n \geq 1$. Find $\liminf_{n \rightarrow \infty} a_n$ and $\limsup_{n \rightarrow \infty} a_n$.

2. Discuss the convergence of the following series: [2+3]

(a) $\sum_{n=1}^{\infty} \log \left(\frac{n}{n+1} \right)$

(b) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

3. Let $\{x_n\}_{n=1}^{\infty}$ be any increasing sequence. Define $y_n = \frac{x_n}{1 + |x_n|}$, $n \geq 1$. [3+2]

(a) Show that the sequence $\{y_n\}_{n=1}^{\infty}$ is increasing.

(b) Determine whether the sequence $\{y_n\}_{n=1}^{\infty}$ is Cauchy.

4. (a) Let $f(x) = \cos \left(\frac{1}{x^2} \right)$. Show that the right limit of $f(x)$ at $x = 0$ do not exist.

(b) Suppose f is continuous function at $x = a$. Then using $\epsilon - \delta$ definition, show that f^2 is continuous at $x = a$. [3+2]