INDIAN INSTITUTE OF TECHNOLOGY DELHI
DEPARTMENT OF MATHEMATICS
SEMESTER I 2020 − 21
MTL 100 (CALCULUS)
Minor examination

DATE: 29/12/2020 Total Marks: 30 Time:10 – 11:30 am

MARKS WILL BE AWARDED ONLY FOR THOSE ANSWERS WITH PROPER JUSTIFICATION

Question 1: Let $(a_n)_{n\geq 1}$, $(b_n)_{n\geq 1}$ be Cauchy sequences. Show that the sequence $(z_n)_{n\geq 1} = (a_1, b_1, a_2, b_2, \ldots, a_n, b_n, \ldots)$ is Cauchy if and only if $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} b_n$. [4]

Question 2: Let $(x_n)_{n\geq 1}$ be a sequence defined by

$$x_1 = 1 \text{ and } x_{n+1} = x_n\left(1 + \frac{\sin n}{2^n}\right), \quad n \geq 1.$$

Discuss the convergence of the sequence $(x_n)_{n\geq 1}$. [4]

Question 3: Check whether the following infinite series are convergent or not:

a)

$$\frac{1^2}{2} + \frac{2^2}{1} + \frac{3^2}{2^2} + \frac{4^2}{2} + \frac{5^2}{2^3} + \frac{6^2}{2^2} + \frac{7^2}{2^4} + \frac{8^2}{2^3} + \cdots$$

b)

$$\sum_{n=1}^{\infty} \frac{(2n)!}{n!\ 3^{2n}}.$$

[2 + 3]

Question 4: Let $f(x) = \begin{cases} \sin(x) & \text{if } x \in [0,\pi] \cap \mathbb{Q}, \\ 0 & \text{if } x \in [0,\pi] \setminus \mathbb{Q}. \end{cases}$
Discuss the continuity of the function f on $[0, \pi]$. [4]

Question 5: Discuss the uniform continuity of the following functions: [3+3]

a) $\sqrt{x} \log x$ on $(0, \infty)$.

b) $\sin(x) \sin(1/x)$ on $(0, 1)$.

Question 6: Does there exist a differentiable function $f : \mathbb{R} \to \mathbb{R}$ such that $f'(0) = 0$ and $f'(x) > 1$ for all $x \neq 0$? Justify your answer. [3]

Question 7: Let $p \in (0, 1)$. Using results of differentiability, show that

$$(1 + x)^p \leq 1 + x^p \quad \text{for all } x > 0.$$

[4]

——ALL THE BEST——