

INDIAN INSTITUTE OF TECHNOLOGY DELHI  
DEPARTMENT OF MATHEMATICS  
MTL 100 (CALCULUS): SEMESTER I 2021 – 22  
Minor

Date: January 9, 2022

Total Marks: 30

Time: 10.00 – 11.15 am

MARKS WILL BE AWARDED ONLY FOR THOSE ANSWERS WITH PROPER JUSTIFICATION

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**Question 1:** Determine whether  $\lim_{n \rightarrow \infty} (n!)^{\frac{1}{n^2}}$  exists. [4]

**Question 2:** Discuss the convergence of the following series: [3+3+4]

$$(a) \sum_{n=1}^{\infty} \frac{n + \log n}{n^3 + n^2 + 1} \quad (b) \sum_{n=2}^{\infty} \frac{(-1)^n}{n \log n} \quad (c) \sum_{n=1}^{\infty} a_n, \text{ where } a_n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{4 \cdot 8 \cdot 12 \cdots (4n)}.$$

**Question 3:** Suppose  $f$  is continuous on  $[0, 2]$  such that  $f(0) = f(2)$ . Show that there exists a  $c \in [0, 1]$  such that  $f(c) = f(c+1)$ . [4]

**Question 4:** Let  $f : [0, 2] \rightarrow \mathbb{R}$  be defined by

$$f(x) = \begin{cases} \cos x & \text{if } x \in [0, 2] \cap \mathbb{Q}, \\ 0 & \text{if } x \in [0, 2] \setminus \mathbb{Q}. \end{cases}$$

Discuss the continuity of  $f$  on  $[0, 2]$ . [5]

**Question 5:** Determine whether the following functions are uniformly continuous: [3+4]

$$(a) f(x) = \cos(1/\sqrt{x}), \quad x \in (0, 1); \quad (b) g(x) = x\sqrt{x}, \quad x \in [0, \infty).$$

—ALL THE BEST—