Department of Mathematics
Indian Institute of Technology Delhi

MAL120 - Mathematics II

Minor - 2: Semester II (2013-14)

Max Time: 1 hour — Total marks: 25

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1. Let $f(z) = \begin{cases} \frac{(\bar{z})^2}{z} & z \neq 0 \\ 0 & z = 0 \end{cases}$. Is f satisfies C-R equation at $z = 0$? Is f differentiable at $z = 0$? Justify your answers. (5)

2. The principal logarithm $\text{Log}(z) := \ln|z| + i\Theta$ for $z \neq 0$, where Θ denotes the argument of z that lies in the interval $(-\pi, \pi]$.

   (a) Prove or disprove that $\text{Log}(z^n) = n\,\text{Log}(z), \quad n \in \mathbb{N}$. (2)

   (b) Show that $\frac{d}{dz}\text{Log}(z) = \frac{1}{z}$ for $-\pi < \Theta < \pi$. (3)

3. Evaluate the integral $\int_C |z|\bar{z}\, dz$ on the path C (positive orientation) which consists of the half- circle $z = Re^{it}$, $0 \leq t \leq \pi$, and the straight line segment: $-R < \text{Re } z < R$, $\text{Im } z = 0$. (3)

4. (a) Let f be a continuous function from $\{z : |z| < 1\}$ into $\mathbb{C}$. Then show that (4)

$$\lim_{r \to 0} \int_{|z|=r} \frac{f(z)}{z} = 2\pi i\, f(0).$$

   (b) Show that $\int_C (z - z_0)^{n-1} = 0 \quad (n = \pm 1, \pm 2, \ldots)$ when C is any closed contour which does not pass through the point z_0. (3)

5. Verify that $\iiint_\Omega \nabla \cdot \vec{F}\, dV = \iint_{\partial\Omega} \vec{F} \cdot \hat{n}\, dS$ where $\vec{F} = (z^2+2)\hat{k}$ and Ω denotes the half of the solid sphere $x^2 + y^2 + z^2 = a^2$ with base $x^2 + y^2 \leq a^2$. (5)

—ALL THE BEST—