Department of Mathematics

MTL 100: Calculus — Total marks: 20
Minor 2: 2017-18 Semester I — Max Time: 1 hr

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1. (a) Let $f : [a, b] \to [a, b]$ be a continuous function. Then prove that there exists a point $x_0 \in [a, b]$ such that $f(x_0) = x_0$.
   (b) Determine the uniform continuity of $g(x) = \tan x$, $x \in (0, \frac{\pi}{2})$. [3+2]

2. Consider the function $f(x) = \sin x$, $x \in (0, \frac{\pi}{4})$. [3+2]
   (a) Find the Taylor polynomial $P_4(x)$ of degree 4 around $x = \frac{\pi}{6}$
   (b) Estimate the error in the above approximation of $f(x)$ by $P_4(x)$ in interval $(0, \frac{\pi}{4})$.

3. Determine the radius of convergence of the following power series: [3+2]

$$(a) \quad \sum_{n=1}^{\infty} \frac{n^{n^2} x^n}{(n+1)^{n^2}} \qquad\qquad (b) \quad \sum_{n=2}^{\infty} \frac{x^n}{\log n}$$

4. Let $f(x, y) = \begin{cases} \dfrac{\sqrt{|y|}}{\sqrt{x^2 + y^2}} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$. [3+2]
   (a) Is f continuous at $(0, 0)$?
   (b) Do partial derivatives f_x and f_y exist at $(0, 0)$? Justify.